

Many cycle lengths in triangle-free graphs with high chromatic number

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Erdős conjectured that a triangle-free graph G with chromatic number $k \geq k_0(\varepsilon)$ contains cycles of at least $k^{2-\varepsilon}$ different lengths as $k \rightarrow \infty$. We prove the stronger fact that every triangle-free graph G with chromatic number $k \geq k_0(\varepsilon)$ contains cycles of $(\frac{1}{64} - \varepsilon)k^2 \log k$ consecutive lengths, and a cycle of length at least $(\frac{1}{4} - \varepsilon)k^2 \log k$. Since there are triangle-free graphs with chromatic number k and at most roughly $4k^2 \log k$ vertices for large k , these results are tight up to a constant factor. We also give new lower bounds on the circumference and the number of different cycle lengths for k -chromatic graphs in other hereditary classes.

This is joint work with B. Sudakov and J. Verstraëte.