Intersection of conjugate solvable subgroups in $GL(n, q)$

Anton Baykalov
The University of Auckland, Auckland, New Zealand
a.baykalov(at)auckland.ac.nz

Assume that a finite group G acts on a set Ω. A point $\alpha \in \Omega$ is G-**regular** if $|\alpha^G| = |G|$, i.e. if the stabilizer of α is trivial. Define the action of the group G on Ω^k by

$$g : (\alpha_1, \ldots, \alpha_k) \mapsto (\alpha_1 g, \ldots, \alpha_k g).$$

If G acts faithfully and transitively on Ω then the minimal number k such that the set Ω^k contains a G-regular point is the **base size** of G and is denoted by $b(G)$. For a positive integer m denote the number of G-regular orbits on Ω^m by $Reg(G, m)$ (this number is 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on Ω. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

In this work we consider Problem 17.41 b) from the "Kourovka notebook" [2]:

Let H be a solvable subgroup of a finite group G that has no nontrivial solvable normal subgroups. Do there always exist five conjugates of H whose intersection is trivial?

The problem is reduced to the case when G is almost simple in [3]. In particular it sufficient to show that for every solvable subgroup $H < G$

$$Reg_H(G, 5) \geq 5$$

for all almost simple groups G.

In [1] the inequality $Reg_H(G, 5) \geq 5$ is shown for almost simple groups with socle isomorphic to an alternating group A_n, $n \geq 5$.

The main statement of the present work is the following

Theorem. *If S is a maximal solvable subgroup of $G = GL(n, q)$; $q \geq 7$, then $Reg_S(G, 5) \geq 5$.*

References

[1] A. A. Baikalov, Intersection of conjugate solvable subgroups in symmetric groups. *Algebra and Logic* **56**(2) (2017) 87–97

[2] V. D. Mazurov, E. I. Khukhro, *Unsolved problems in group theory. The Kourovka notebook* **18**. URL: http://arxiv.org/abs/1401.0300

[3] E. P. Vdovin, On the base size of a transitive group with solvable point stabilizer. *J. Algebra Appl.* 11 (2012) **1** 1250015, 14 pp.