Sobolev Institute of Mathematics
Novosibirsk State University

Graphs and Groups, Representations and Relations

Novosibirsk, Russia, August, 06 – 19, 2018

Abstracts

Novosibirsk – 2018

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Contents

General Information

Venue:
The International Conference and PhD-Master Summer School on "Graphs and Groups, Representations and Relations" (G2R2), August 06 – 19, 2018, Novosibirsk, Akademgorodok, Russia.

The main goal:
To bring together researchers from different mathematical fields including graph theory, group theory, low dimensional geometry, topology, and their applications.

Scientific Programme committee:
Alexander Gavrilyuk, Alexander A. Ivanov, Elena Konstantinova (*co-chair*), Denis Krotov, Alexander Mednykh (*co-chair*), Andrey Vasil'ev, Yaokun Wu.

Organizing committee:
Nikolay Abrosimov, Sergey Goryainov, René van Bevern, Kristina Kaushan, Ekaterina Khomyakova, Sergey Konstantinov, Elena Konstantinova (*chair*), Alexey Medvedev, Ilya Mednykh, Kristina Rogalskaya, Grigory Ryabov, Anna Simonova, Ev Sotnikova, Ivan Takhonov, Tatyana Yakovleva.

Steering committee:
Sergey Goryainov, Elena Konstantinova, Klavdija Kutnar, Alexander Makhnev, Natalia Maslova, Alexander Mednykh.

Organizers:
Sobolev Institute of Mathematics, Novosibirsk, Russia
Novosibirsk State University, Novosibirsk, Russia

Partners:
Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg, Russia

Sponsors:
Regional Mathematical Center, Novosibirsk State University, Novosibirsk, Russia
Project 5-100, Novosibirsk State University, Novosibirsk, Russia

Website:
`math.nsc.ru/conference/g2/g2r2`

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About G2

G2-events are International Conferences and PhD–Master Summer Schools on Graphs, Groups, and related topics. The main goal of the events is to bring together students, young researchers, scientists, and experts to exchange knowledge and results in a broad range of topics relevant to graph theory and group theory with connections to combinatorics, topology, geometry, coding theory, automata and formal language theory, algorithm theory, network analysis, and applications.

The first event of the series, "Graphs and Groups, Cycles and Coverings" (G2C2), was held on September 23 – 26, 2014, in Akademgorodok, Novisibirsk, Russia, in the frame of the International cooperation between Slovenia and Russia in 2014 – 2015 with the support of the Slovenian Research Agency. Five slovenian mathematicians from University of Primorska, Koper, presented excellent talks on group actions on combinatorial objects. The talks were given by Klavdija Kutnar, Tomaž Pisanski, Aleksander Malnič, István Kovács, and Istvăn Estělyi. It was a small workshop with no more than 30 participants from Novosibirsk, Moscow, Yekaterinurg, Chelyabinsk, however, it was successful enough. It was organized by Sobolev Institute of Mathematics and Novosibirsk State University. The main organizers of the workshop were Elena Konstantinova, Klavdija Kutnar, and Alexander Mednykh.

After a success of G2C2, the series of G2–events was conceived by the steering committee which includes Sergey Goryainov, Elena Konstantinova, Klavdija Kutnar, Alexander Makhnev, Natalia Maslova, and Alexander Mednykh. In the winter of 2015 it was decided to have the next event in Yekaterinburg.

The second event, The International Conference and PhD Summer School "Groups and Graphs, Algorithms and Automata" (G2A2), was organized by Krasovskii Institute of Mathematics and Mechanics and Ural Federal University named after the first President of Russia B.N. Yeltsin with Alexander Makhnev as the chair of the scientific committee. The main organizers of G2A2 were Vladislav Kabanov, Mikhail Volkov, Natalia Maslova and Sergey Goryainov. It was held on August 9 – 15, 2015, in the recreation area Ivolga which is located near Yekaterinburg, Russia. The topic of the event included all the branches of group theory, graph theory, automata and formal language theory, and algorithm theory. The scientific program of G2A2 event consisted of minicourses, plenary and contributed talks. Klavdija Kutnar and Dragan Marušič presented the exciting course on Graphs and their Automorphism Groups. Tomaž Pisanski teached students on Symmetries in Graphs with Python and Sage. After taking that course many students started to use Python as principal software to write papers, reports, etc. With a great enthusiasm, Mikhail Volkov explained a problem on Synchronizing finite automata which everyone can understand but nobody can solve (so far). The team of main speakers was presented by Bernhard Amberg (Johannes Gutenberg University Mainz), Tatsuro Ito (Anhui University), Lev Kazarin (Demidov Yaroslavl State University), Jack Koolen (University of Science and Technology of China), Vladimir Levchuk (Siberian Federal University), Nadezhda Timofeeva (Demidov Yaroslavl State University), Evgeny Vdovin (Sobolev Institute of Mathematics). Around 100 experts on finite group theory, graph theory, algebraic combinatorics, automata and formal language theory, and algorithm theory from 7 countries (Belarus, China, Germany, Hungary, Slovenia, Russia, and Ukraine) participated in the G2A2–event.

The third event, The International Conference and PhD–Master Summer School on "Graphs and Groups, Spectra and Symmetries" (G2S2), was held on August 15 – 28, 2016, in Novosibirsk, Russia. The G2S2–event was organized by Sobolev Institute of Mathematics and Novosibirsk State University with cooperation of the Krasovskii Institute of Mathematics and Mechanics, and with Elena Konstantinova, Denis Krotov, and Alexander Mednykh as main organizers. The G2S2–event was supported by the Russian Foundation for Basic Research, grant 16 – 31 – 10290, and Novosibirsk State University, Project 5 – 100. More than 110 experts on finite group theory, graph theory, algebraic combinatorics, low–dimensional geometry and topology from 19 countries (Brazil, Canada, China, Czech Republic, Finland, Germany, Hungary, India, Iran, Israel, Italy, Japan, Slovakia, Slovenia, South Korea, Taiwan, United Kingdom, USA and Russia) participated in the G2S2–events. Young scientists, PhD students, graduate and undergraduate students were presented by 75 participants. The scientific program of G2S2–events consisted of minicourses, plenary and contributed talks, open problems session. Four minicourses, each containing eight 50–minutes lectures, were given by Lih-Hsing Hsu (Providence University, Taichung, Taiwan), Bojan Mohar (University of Ljubljana, Slovenia; Simon Fraser University, Vancouver, Canada), Alexander A. Ivanov (Imperial College London, UK) and Ted Dobson (Mississippi State University, USA; University of Primorska, Slovenia). Twenty main speakers gave brilliant talks on algebraic combinatorics, on isomorphism problem for graphs, Cayley graphs and Cayley combinatorial objects, on colour–

preserving automorphisms of Cayley graphs, on integral graphs and Cayley graphs, on characterization of the Grassmann graphs and on applications of Hoffman graphs, on symmetry properties of combinatorial objects, on the classification of association schemes, on codes obtained from some graphs and finite geometries, on partial geometry with given parameters, on topological graph theory problems, on the lit-only sigma game and some mathematics around, on plateaued Boolean functions with the same spectrum support. As it was noticed by Alexander A. Ivanov, the source of many ideas discussed during the conference was Com^{2}Mac, The Combinatorial and Computational Mathematics Center, POSTECH, Korea (1999 – 2008), supervising by Professor Jin Ho Kwak (Beijing Jiaotong University, China), who participated at the G2S2–events. 15 participants of G2S2 were the members of this institution for a while. The list of the main speakers is given by Anton Betten, Shaofei Du, Alexander Gavrilyuk, Mitsugu Hirasaka, Tatsuro Ito, Lev Kazarin, Jack Koolen, Klavdija Kutnar, Jin Ho Kwak, Dragan Marušič, Akihiro Munemasa, Mikhail Muzychuk, Roman Nedela, Patric Patric Östergård, Ilia Ponomarenko, Yuriy Tarannikov, Andrey Vasil'ev, Yaokun Wu, Matan Ziv-Av. The participants of the event were invited to submit a research paper based on the talks for the proceedings to appear as a Special Issue of the Siberian Electronic Mathematical Reports (SEMR). Totally, 12 papers were published in SEMR in 2016 (see `http://math.nsc.ru/conference/g2/g2s2/papers.html`). To find more details on G2S2–event, see [1]. Video lectures of all courses and plenary talks are published online by Youtube, see `http://math.nsc.ru/conference/g2/g2s2/video.html`.

The fourth event, The International Conference and PhD-Master Summer School on "Groups and Graphs, Metrics and Manifolds" (G2M2), was held on July 22 – 30, 2017, in Yekaterinburg, Russia. The G2M2–event was organized by Krasovskii Institute of Mathematics and Mechanics and Ural Federal University named after the first President of Russia B.N. Yeltsin with Alexander Makhnev as chair and Sergey Matveev as co-chair of the scientific committee, and Vladislav Kabanov as chair and Mikhail Volkov as co-chair of the organizing committee. The scientific G2M2–program included minicourses, plenary and contributed talks. The brilliant course on "Introduction to the character theory of finite groups of Lie type" was given by Alexandre Zalesski. Significance of the theory for group theory derives from the fact that the majority of simple groups are groups of Lie type, so one cannot ignore these groups when approaching problems of general nature. The aim of the lectures was to focus on the key elements of the theory in order to help beginners to orient in the area and understand the central ideas of the theory. A strong version of the Sims conjecture on finite primitive permutation groups was discussed in the minicourse given by Anatoly Kondrat'ev. A very nice review on old results obtained by Sims (1967), Thompson (1970), Wielandt (1971), Knapp (1973, 1981), Fomin (1980), Cameron, Praeger, Saxl and Seitz (1983) was given along with an excellent explanation of a strengthened version of the Sims conjecture which is based on graph-theoretical approach and associated with the automorphism group acting primitively on vertex set of an undirected connected finite graph. The version was formulated by Vladimir Trofimov and the lecturer in 1999. In this course a more general problem than strengthened version of the Sims conjecture was also presented. The aim of the lectures was to discuss obtained results and some methods of their proofs. The list of the main conference speakers was presented by 15 plenary talks given by experts from finite group theory, algebraic combinatorics, coding theory, topological graph theory, including Shaofei Du, Tatsuro Ito, Alexander A. Ivanov, Denis Krotov, Alexander Makhnev, Alexander Mednykh, Akihiro Munemasa, Danila Revin, Sergey Shpectorov, Vladimir Trofimov, Andrey Vasil'ev, Mikhail V. Volkov, Yaokun Wu. To find more details on G2M2–event, visit `http://g2.imm.uran.ru/g2m2/index.html`.

The current event, The International Conference and PhD-Master Summer School on "Graphs and Groups, Representations and Relations" (G2R2) is organized by Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia. G2R2 is about strong and beautiful mathematics including graph theory, group theory, low dimensional geometry, topology, and their applications. PhD-Master Summer School presented by 4 courses and 40 students from 11 countries is included into the program of Siberian Summer Schools (`https://education.nsu.ru/summerschools_math/`). The scientific G2R2-conference program consists of 21 plenary talks, 48 contributed talks and open problems session.

We are looking forward to have you as one of the participants of G2R2–events.

Enjoy the art of mathematics with us!

References

[1] E. V. Konstantinova, D. S. Krotov, A. D. Mednykh, On Graphs and Groups, Spectra and Symmetries held on August 15-28, 2016, Novosibirsk, Russia. *Sib. Electron. Mat. Rep.* **13** (2016) 1369–1382.

Conference Program

All Conference activities and lectures of Summer School minicourses take place in the Novosibirsk State University, Pirogova 1, Conference room 3107.

Classes of minicourses take place in the Novosibirsk State University, Pirogova 1, Seminar room 4109.

Monday, August 06

09:00 - 18:00	Registration: *Room* 4105
18:00 - 19:00	Excursion: NSU dome
19:00 - 21:00	Welcome party: *Hall of the Conference room* 3107

Tuesday, August 07

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
	Chair: Tomaž Pisanski
10:00 - 10:50	Roman Nedela: *Lecture 1*
11:00 - 11:50	Roman Nedela: *Lecture 2*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
	Chair: Alexander Mednykh
12:10 - 13:00	Gareth Jones: *Lecture 1*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Gareth Jones: *Lecture 2*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Akihiro Munemasa
16:00 - 16:25	Alexander Gavrilyuk: *On triple intersection numbers of association schemes*
16:30 - 16:55	Keiji Ito: *Second eigenmatrices of a non-commutative association schemes obtained from steiner systems*
17:00 - 17:25	Yan Zhu: *Relative t-designs on one shell of Johnson association schemes*
17:30 - 17:55	Ivan Mogilnykh: *On equitable* 2*-partitions of Hamming graphs* $H(n, q)$ *with eigenvalues* λ_2
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Da Zhao: *Group fusion power of association schemes*
18:35 - 19:00	Katie Brodhead: *Label classification in machine learning via association schemes*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 1**
	Chair: Nikolay Abrosimov

Wednesday, August 08

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
	Chair: Tomaž Pisanski
10:00 - 10:50	Roman Nedela: *Lecture 3*
11:00 - 11:50	Roman Nedela: *Lecture 4*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
	Chair: Alexander Mednykh
12:10 - 13:00	Gareth Jones: *Lecture 3*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Gareth Jones: *Lecture 4*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: István Kovács
16:00 - 16:25	Nikolai Erokhovets: *Construction of fullerenes and Pogorelov polytopes*
16:30 - 16:55	Young Soo Kwon: *Classification of t-balanced regular Cayley maps on some groups*
17:00 - 17:25	Zeying Xu: *Tropical hyperplane arrangement and zonotopal tiling*
17:30 - 17:55	Andrei Tetenov: *Inverse limits of m-sprouts and topological self-similar dendrites*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Marina Chanchieva: *The properties of the set of subarcs of a symmetric irrational dendrite*
18:35 - 19:00	Dmitry Drozdov: *Deformations of polygonal dendrites in the plane*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 2**
	Chair: Alexander Mednykh

Thursday, August 09

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
	Chair: Tatsuro Ito
10:00 - 10:50	Alexander A. Ivanov: *Locally Projective Graphs of $GF(2)$-type*
11:00 - 11:50	Mario Mainardis: *Recent developments in Majorana representations of the symmetric groups*
11:50 - 12:10	*Coffee break*
	Chair: Alexander A. Ivanov
12:10 - 13:00	Atsushi Matsuo: *Around symmetries of vertex operator algebras*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Jiping Zhang: *Character degree graphs of finite groups*
15:20 - 16:00	*Coffee break*
	Chair: Leonard H. Soicher
16:00 - 16:50	István Kovács: *Skew-morphisms and regular Cayley maps for dihedral groups*
17:00 - 17:50	Štefan Gyürki: *On directed strongly regular graphs*
17:50 - 18:10	*Coffee break*
18:10 - 19:00	Louis H. Kauffman: *Counting colorings of cubic graphs via a generalized Penrose bracket*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Volleyball: NSU Sports Complex**

Friday, August 10

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
	Chair: Tomaž Pisanski
10:00 - 10:50	Roman Nedela: *Lecture 5*
11:00 - 11:50	Roman Nedela: *Lecture 6*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
	Chair: Alexander Mednykh
12:10 - 13:00	Gareth Jones: *Lecture 5*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Gareth Jones: *Lecture 6*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Jiping Zhang
16:00 - 16:25	Maria Zvezdina: *On the splitness of the prime and solvable graphs for finite simple groups*
16:30 - 16:55	Anton Baykalov: *Intersection of conjugate solvable subgroups in* $GL(n,q)$
17:00 - 17:25	Grigory Ryabov: CI*-property for decomposable Schur rings over an elementary abelian group*
17:30 - 17:55	Eun-Kyung Cho: *On groups whose proper Schur rings are commutative*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Ilya Gorshkov: *On a connection between the order of a finite group and the set of conjugacy classes size*
18:35 - 19:00	Stepan Fadeev: *The study of energy configurations in the Thomson problem*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 1**
	Chair: Nikolay Abrosimov

Saturday, August 11

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 1
	Chair: Tomaž Pisanski
10:00 - 10:50	Roman Nedela: *Lecture 7*
11:00 - 11:50	Roman Nedela: *Lecture 8*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 2
	Chair: Alexander Mednykh
12:10 - 13:00	Gareth Jones: *Lecture 7*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Gareth Jones: *Lecture 8*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Edwin van Dam
16:00 - 16:25	Vladislav Kabanov: *Prolific construction of strictly Deza graphs*
16:30 - 16:55	Rhys Evans: *Neumaier graphs*
17:00 - 17:25	Dmitry Panasenko: *On equitable partitions of divisible design graphs*
17:30 - 17:55	Konstantin Vorob'ev: *Alphabet lifting construction of equitable partitions of Hamming graphs*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Faina Solov'eva: *On uniform partitions of* F^n *into Hamming codes*
18:35 - 19:00	Ismael Gonzalez Yero: *Graphs and metrics: the partition case*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 2**
	Chair: Alexander Mednykh

Sunday, August 12

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
	Chair: Mario Mainardis
10:00 - 10:50	Leonard H. Soicher: *Cliques and colourings in GRAPE*
11:00 - 11:50	Sergey Goryainov: *Several results on cliques in strongly regular graphs*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Edwin van Dam: *Applications of semidefinite programming, symmetry, and algebra to graph partitioning problems*
13:00 - 14:30	*Lunch*
	Chair: Alexander Mednykh
14:30 - 15:20	Tomaž Pisanski: *Symmetries and combinatorics of finite antilattices*
15:20 - 16:00	*Coffee break*
16:00 - 16:50	Victor Buchstaber: *Cyclically* 5*-edge connected graphs, fullerenes and Pogorelov polytopes*
17:00 - 17:30	*Conference photo*
19:00 - 22:00	*Conference dinner*

Monday, August 13

10:00 - 11:00	Excursion: NSU dome
10:00 - 20:00	City Tour
11:30 - 13:00	**Football: NSU Sports Complex**

Tuesday, August 14

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
	Chair: Alexander A. Ivanov
10:00 - 10:50	Akihiro Munemasa: *Lecture 1*
11:00 - 11:50	Akihiro Munemasa: *Lecture 2*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
	Chair: Andrey Vasil'ev
12:10 - 13:00	Mikhail Muzychuk: *Lecture 1*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Mikhail Muzychuk: *Lecture 2*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Atsushi Matsuo
16:00 - 16:25	Danila Revin: *Integrality of some Cayley graphs*
16:30 - 16:55	Honghai Li: *On the spectra of Cayley graphs*
17:00 - 17:25	Alexandr Valyuzhenich: *On eigenfunctions of Hamming graphs with minimum support*
17:30 - 17:55	Yuefeng Yang: *Weakly distance-regular digraphs*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Yi Dai: *Kernels in* 3*-regular circulant digraphs*
18:35 - 19:00	Ludmila Tsiovkina: *On vertex-transitive antipodal distance-regular graphs of diameter three with primitive almost simple antipodal groups*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 3**
	Chair: Alexandr Valyuzhenich

Wednesday, August 15

07:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
	Chair: Alexander A. Ivanov
10:00 - 10:50	Akihiro Munemasa: *Lecture 3*
11:00 - 11:50	Akihiro Munemasa: *Lecture 4*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
	Chair: Danila Revin
12:10 - 13:00	Mikhail Muzychuk: *Lecture 3*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Mikhail Muzychuk: *Lecture 4*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Tatsuro Ito
16:00 - 16:25	Vladimir N. Potapov: *Construction of pairs of orthogonal latin cubes based on combinatorial designs*
16:30 - 16:55	Ivan Mogilnykh: *On regular subgroups of the automorphism group of the Hamming code of length* 15
17:00 - 17:25	Ev Sotnikova: *Minimum supports of eigenfunctions in bilinear forms graphs*
17:30 - 17:55	Anna Taranenko: *On the numbers of transversals and multiplexes in iterated quasigroups*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Maria Lisitsyna: *On perfect* 2*-colorings of infinite multipath graphs*
18:35 - 19:00	Olga Parshina: *Perfect* 2*-colorings of infinite circulant graphs with a continuous set of odd distances*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 4**
	Chair: Grigory Ryabov

Thursday, August 16

07:30 - 09:45	*Breakfast*
	Conference: Invited Talks
	Chair: Yaokun Wu
10:00 - 10:50	Jack Koolen: *Recent progress on graphs with fixed smallest eigenvalue*
11:00 - 11:50	Vsevolod Gubarev: *PC-polynomial on graph and its largest root*
11:50 - 12:10	*Coffee break*
	Chair: Danila Revin
12:10 - 13:00	Leonid Bokut: *Groebner-Shirshov bases for groups, semigroups, categories, and Lie algebras*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Evgeny Vdovin: *On* 2*-closures of primitive solvable permutation groups*
15:20 - 16:00	*Coffee break*
	Chair: Roman Nedela
16:00 - 16:50	Sergey Lando: *Delta-matroids and Vassiliev invariants*
17:00 - 17:50	Norman Wildberger: *Combinatorial games on graphs, Coxeter-Dynkin diagrams, and the geometry of root systems*
17:50 - 18:10	*Coffee break*
18:10 - 19:00	**Conference: Open problems session**
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Volleyball: NSU Sports Complex**

Friday, August 17

08:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
	Chair: Alexander A. Ivanov
10:00 - 10:50	Akihiro Munemasa: *Lecture 5*
11:00 - 11:50	Akihiro Munemasa: *Lecture 6*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
	Chair: Andrey Vasil'ev
12:10 - 13:00	Mikhail Muzychuk: *Lecture 5*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Mikhail Muzychuk: *Lecture 6*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Norman Wildberger
16:00 - 16:25	Jan Kim: *Non-residually finite direct limits of the* 2*-Bridge link groups*
16:30 - 16:55	Alexander Mednykh: *Arithmetics and combinatorics of circulant graphs*
17:00 - 17:25	Sam Mattheus: *Polarities of projective planes and related (hyper) graphs*
17:30 - 17:55	Bao Vuong: *The volume of a compact hyperbolic antiprism*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Michele Mulazzani: *Compact n-manifolds via $(n+1)$-colored graphs*
18:35 - 19:00	Kirill Kamalutdinov: *On the semigroup of similarities with unique one point intersection, not satisfying WSP*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 3**
	Chair: Alexandr Valyuzhenich

Saturday, August 18

08:30 - 09:45	*Breakfast*
	PhD-Master Summer School: Minicourse 3
	Chair: Alexander A. Ivanov
10:00 - 10:50	Akihiro Munemasa: *Lecture 7*
11:00 - 11:50	Akihiro Munemasa: *Lecture 8*
11:50 - 12:10	*Coffee break*
	PhD-Master Summer School: Minicourse 4
	Chair: Andrey Vasil'ev
12:10 - 13:00	Mikhail Muzychuk: *Lecture 7*
13:00 - 14:30	*Lunch*
14:30 - 15:20	Mikhail Muzychuk: *Lecture 8*
15:20 - 16:00	*Coffee break*
	Conference: Contributed talks
	Chair: Alexander Mednykh
16:00 - 16:25	Huye Chen: *On edge-transitive factorizations of complete uniform hypergraphs*
16:30 - 16:55	Yanzhen Xiong: *Competition numbers and phylogeny numbers*
17:00 - 17:25	Chengyang Qian: *A simple multibody system on a discrete circle*
17:30 - 17:55	Yinfeng Zhu: *Phase spaces and kernel spaces of transformation semigroups*
18:00 - 18:10	*Coffee break*
18:10 - 18:35	Yaokun Wu: *An expansion property of Boolean multilinear map*
18:35 - 19:00	Ilya Mednykh: *Jacobian and complexity of I-graphs*
19:00 - 20:00	*Dinner*
20:00 - 22:00	**Problem solving: Minicourse 4**
	Chair: Grigory Ryabov

Sunday, August 19

08:30 - 09:45	*Breakfast*
	Conference: Invited Talks
	Chair: Elena Konstantinova
10:00 - 10:50	Tatsuro Ito: *The Terwilliger algebra of a tree*
11:00 - 11:50	Bobo Hua: *Combinatorial curvature for infinite planar graphs*
11:50 - 12:10	*Coffee break*
12:10 - 13:00	Alexander Gavrilyuk: *On mixed Moore-Cayley graphs*
13:00 - 13:30	*Conference photo*
	Closing

Abstracts

Abstracts of Minicourses, Plenary and Contributed talks are listed alphabetically with respect to the Presenting Author

Minicourses

Minicourse I: Graph coverings and harmonic morphisms between graphs

Lecturer:
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A covering between two graphs is a graph epimorphism which is locally bijective. Although the concept of coverings of topological spaces was well known in algebraic topology for a long time, the systematic combinatorial approach to graph coverings is related to the solution of the Heawood map colouring problem by Ringel and Youngs. Nowadays the concept of graph coverings forms an integral part of graph theory and has found dozen of applications, in particular, as a strong construction technique. The aim of the course is to explain foundations of the combinatorial theory of graph coverings and its extension to branched coverings between 1-dimensional orbifolds.

In the course we shall follow the attached plan.

Part 1. Graphs and Groups: graphs with semi-edges and their fundamental groups; actions of groups on graphs; highly symmetrical graphs; subgroup enumeration in some finitely generated groups; enumeration of conjugacy classes of subgroups.

Part 2. Graph coverings and voltage spaces: graph coverings and two group actions on a fibre; voltage spaces; permutation voltage space; Cayley voltage space, Coset voltage space; equivalence of coverings and T-reduced voltage spaces; enumeration of coverings.

Part 3. Applications of graph coverings: regular graphs with large girth; large graphs of given degree and diameter; nowhere-zero flows and coverings; 3-edge colourings of cubic graphs; Heawood map coloring problem.

Part 4. Lifting automorphism problem: classical approach; lifting of graph automorphisms in terms of voltages; lifting problem - case of abelian CT(p); case of elementary abelian CT(p).

Part 5. Branched coverings of graphs: definition and basic properties; Riemann Hurwitz theorem for graphs; Laplacian of a graph and the Matrix-Tree Theorem; Jacobians and harmonic morphisms; graphs of groups, uniformisation.

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[2] A. Mednykh, R. Nedela, Harmonic morphisms of graphs, Part I: Graph Coverings, Matej Bel University, 2015.

Minicourse II: Groups and symmetry in low dimensional geometry and topology

Lecturer:
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The course considers some applications of group theory to geometry in dimensions 2 and 3. The main theme will be the study of the automorphism groups of compact Riemann surfaces, especially those surfaces uniformised by subgroups of finite index in triangle groups. By Belyi's Theorem, these are the compact Riemann surfaces which, when regarded as complex algebraic curves, can be defined over an algebraic number field. As such, they give a faithful representation of the absolute Galois group (the automorphism group of the field of algebraic numbers), a group of great complexity and importance in algebraic geometry. These surfaces include the Hurwitz surfaces, those attaining Hurwitz's upper bound of 84(g-1) for the size of the automorphism group of a compact Riemann surface of genus g>1. The course will consider the corresponding Hurwitz groups, the finite quotients of the (2,3,7) triangle group, and it will conclude with a brief look at the corresponding situation in dimension 3, where the normaliser of the Coxeter group [3,5,3] plays a similar role.

Outline of the course:

Lecture 1. Riemann surfaces and Fuchsian groups.

Lecture 2. Compact Riemann surfaces and their automorphism groups.

Lecture 3. Triangle groups and their quotients.

Lecture 4. Maps and hypermaps on surfaces.

Lecture 5. Dessins d'enfants, and Belyi's Theorem.

Lecture 6. The absolute Galois group, and its action on dessins.

Lecture 7. Hurwitz groups and surfaces.

Lecture 8. Hyperbolic 3-manifolds with large symmetry groups.

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Minicourse III: Permutation representations of finite groups and association schemes

Lecturer:
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In these lectures, we first introduce the theory of permutation representations of finite groups. The existence of a canonical basis of a permutation module makes it different from a general module, leading to numerical invariants such as Krein parameters. Krein parameters are an analogue of tensor product coefficients for irreducible representations, as seen by Scott's theorem. We then discuss multiplicity-free permutation representations in detail, giving a motivation to a more general concept of commutative association schemes. Lack of a group in the definition leads to slight discrepancy in theory, and a long standing conjecture about splitting fields.

Outline of the course:

Lecture 1. Transitive permutation groups and orbitals.

Lecture 2. Permutation modules and the centralizer algebra.

Lecture 3. Spherical functions and eigenvalues.

Lecture 4. The holomorph of a group.

Lecture 5. Krein parameters and Scott's theorem.

Lecture 6. Association schemes as an abstract centralizer algebra.

Lecture 7. Eigenmatrices of association schemes.

Lecture 8. Splitting fields of association schemes.

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Minicourse IV: Coherent configurations and association schemes: structure theory and linear representations

Lecturer:
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The goal of my lectures is to give an introduction to the theory of coherent configurations with the main focus on a particular case of association schemes. The closely related objects like Schur rings and table algebras will be presented too. In my lectures I will talk about the structure and representation theories of association schemes. A connection between coherent configurations and permutation groups known as Galois correspondence will be discussed too. Some classical results and new developments in this area with their applications will be presented. I also will remind and discuss some open problems in this area.

Outline of the course:

Lectures 1-2-3. Coherent configurations. Association schemes. Schur rings. Table algebras (main definitions and basic properties).

Lecture 4. Galois correspondence between coherent configurations and permutation groups. Schurian coherent configurations and 2-closed permutation groups.

Lecture 5-6. Representation theory of coherent configurations (the semisimple case). Frame number. Applications of representation theory.

Lecture 7. Structure theory of association schemes. Closed subsets, quotients, normal and strongly normal closed subsets.

Lecture 8. Primitive association schemes.

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Plenary Talks

Groebner-Shirshov bases for groups, semigroups, categories, and Lie algebras

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This is joint work with Yuqun Chen

Groebner bases and Groebner-Shirshov bases were invented independently in 1960th by A.I. Shirshov for ideals of free (commutative, anti-commutative) non-associative algebras, free Lie algebras, and implicitly free associative algebras, by H. Hironaka for ideals of the power series algebras (both formal and convergent), and by B. Buchberger for ideals of the polynomial algebras. Groebner bases and Groebner-Shirshov bases theories have been proved to be useful in different branches of mathematics, including commutative algebras; non-commutative and non-associative (super-, Lie, Kac-Moody, Drinfeld quantum group, conformal, ...) algebras; modules; semigroups (groups, categories, semirings,...) via semigroup (group, category, semiring, ...) algebras; operads; etc.. It is a powerful tool to study the classical subjects in algebra: relations; representations; free linear universal algebras; normal forms; word problems; conjugacy problems; algorithmic problmsconfluence rewriting systems; automaton groups and semigroups; embedding theorems; PBW type theorems; extensions; homologies; growth functions; Dehn functions; complexities; etc. We will emphasize on Groebner - Shirsov bases and normal forms for finite Coxeter and braid groups, plactic monoid, simplicial and cyclic categories, semisimple Lie algebras, Shirshov-Cartier-Cohn counter examples to PBW theorem for Lie algebras over a commutative algebra.

Composition-Diamond Lemma for associative algebras over a field. Let S be a monic subset of an algebra $k<X>$ (free associative), and $(X*, \leq)$ be the deg-lex order of on words $X*$. Then the following conditions are equivalent:

(i) S is a GS basis (any composition $(f,g)_w = fb - -ag$, where $w = \bar{f}b = a\bar{g}$, $\bar{f}$ be the maximal word in f, is “trivial” $mod(S,w)$;

(ii) $f \in Ideal(S) \longrightarrow \bar{f} = u\bar{s}v$ for some $s \in S$;

(iii) $Irr(S) = \{w, w \neq u\bar{s}v$ for any $s \in S\}$ is a linear basis of $k<X|S>= k<X>/Ideal(S)$.

Cyclically 5-edge connected graphs, fullerenes and Pogorelov polytopes

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In this talk, we discuss fruitful connections between classical and recent results of the graph theory, the polytope theory, hyperbolic geometry and algebraic topology.

A 3-valent planar 3-connected graph is *cyclically* 5-*edge connected* (c5-*connected*) if it has at least 5 vertices and no two circuits can be separated by cutting fewer than 5 edges. A graph is *strongly cyclically* 5-*edge connected* (c^*5-*connected*) if in addition any separation of the graph by cutting 5 edges leaves one component that is a simple circuit of 5 edges. These notions are well-known and play an important role in the graph theory. By the result of G. D. Birkhoff (1913), the famous Four Colour Theorem for planar graphs can be reduced to the class of c^*5-connected graphs. In 1974 D. Barnette and J. W. Butler shown independently that any c5-connected graph can be obtained from the graph of dodecahedron by a simple set of operations. An analogous description for c^*5-connected graphs was found by D. Barnette in 1977. Later a part of this result was rediscovered by T. Inoue (2008) in the context of hyperbolic geometry.

There is a remarkable geometric characterisation of c5-connected graphs due to A. V. Pogorelov (1967) and E. M. Andreev (1970): a combinatorial 3-polytope can be realised in Lobachevsky space as a bounded polytope with right dihedral angles if and only if its graph is c5-connected. We refer to such combinatorial polytopes as *Pogorelov polytopes* ($\mathcal{P}$-*polytopes*). Generalising the classical construction of Löbell (1931), A. Yu. Vesnin in 1987 described a way to produce a hyperbolic 3-manifold from any Pogorelov polytope by endowing it with an additional structure related to the hyperbolic reflection group (this structure consists of $\mathbb{Z}/2$-vectors assigned to the facets of the polytope). An important example of this additional structure arises from the Four Colour Theorem. A. Yu. Vesnin also conjectured that hyperbolic manifolds arising from 4-colourings of one special series of Pogorelov polytopes (the so-called *Löbell polytopes* or *barrels*) are isometric if and only the 4-colourings are equivalent. In 2017 V. M. Buchstaber, N. Yu. Erokhovets, M. Masuda, T. E. Panov and S. Park proved that hyperbolic manifolds arising from any Pogorelov polytopes are isometric if and only if the polytopes with additional structures are combinatorially equivalent. Using this result V. M. Buchstaber and T. E. Panov proved that hyperbolic manifolds arising from 4-colourings of any Pogorelov polytopes are isometric if and only if the colourings are equivalent, thereby verifying Vesnin's conjecture.

According to T. Doslic (2003), the class of $\mathcal{P}$-polytopes contains fullerenes, i.e. simple 3-polytopes with only 5- and 6-gonal faces. V. M. Buchstaber and N. Yu. Erokhovets (2017) obtained the results describing the class of $\mathcal{P}$-polytopes constructively:

(1) Any $\mathcal{P}$-polytope except for the k-barrels can be obtained from the 5 or the 6-barrel by a sequence of two-edges-truncations and connected sums with 5-barrels along 5-gons.

(2) Any fullerene except for the 5-barrel and the $(5, 0)$-nanotubes can be obtained from the 6-barrel by a sequence of $(2, 6; 5, 5)$-, $(2, 6; 5, 6)$-, $(2, 7; 5, 6)$-, $(2, 7; 5, 5)$-truncations such that all intermediate polytopes are either fullerenes or $\mathcal{P}$-polytopes with facets 5-, 6- and at most one additional 7-gon adjacent to a 5-gon.

The smallest eigenvalues of Hamming, Johnson and other graphs

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This is joint work with Andries Brouwer, Ferdinand Ihringer and Matt McGinnis

If S is a subset of vertices of a graph $G = (V, E)$, then $e(S, \overline{S})$ denotes the number of edges of G with exactly one endpoint in S. The MAX CUT problem is the problem of determining $mc(G) := \max_{S\subset V} e(S, \overline{S})$ over all proper subsets S of V which is the same as finding a bipartite subgraph of G with the maximum number of edges. This is a well known NP-hard problem, but in a 1995 breakthrough work, Goemans and Williamson used semidefinite programming techniques to obtain an approximation algorithm for MAX CUT with performance ratio at least $\alpha = \frac{2}{\pi}\min_{0<\theta\leq\pi}\frac{\theta}{1-\cos\theta}$ $(0.85856 < \alpha < 0.85857)$. Their work uses a semidefinite relaxation of the inequality $mc(G) \leq n\mu_{max}(G)/4$, where $\mu_{max}(G)$ equals the largest eigenvalue of the Laplacian matrix of G. If G is d-regular, this inequality becomes $mc(G) \leq n(d - \lambda_{min})/4$.

In 1999, Karloff proved that the performance ratio of the Goemans-Williamson algorithm is exactly α and showed that it is impossible to add valid linear constraints to improve this performance ratio. Karloff's work hinged on determining the smallest eigenvalue of graphs in the Johnson association scheme. Karloff determined the smallest eigenvalue of these graphs in a certain range of parameters and made a conjecture about a larger set of parameters where his result should hold. The Johnson graph $J(n, d, j)$ is the graph whose vertices are the d-subsets of a set with n elements where two vertices are adjacent when they meet in a $(d - j)$-set. Delsarte showed that the eigenvalues of the $J(n, d, j)$ are given by the Eberlein polynomials $E_j(i) = \sum_{h=0}^{i}(-1)^{i-h}\binom{i}{h}\binom{d-h}{j}\binom{n-d-i+h}{n-d-j}$ for $0 \leq i \leq d$.

Conjecture 1. [Karloff 1999] *Let $n = 2d$ and $j > d/2$. Then the smallest eigenvalue of $J(n, d, j)$ is $E_j(1)$.*

In 2000, Alon and Sudakov applied the relation between $mc(G)$ and λ_{min} and extended Karloff's results. Their work relied heavily on determining the smallest eigenvalue of certain graphs in the Hamming association scheme. In 2016, Van Dam and Sotirov used semidefinite programming techniques to study the MAX k-CUT program (maximizing the number of edges in a k-partite subgraph of a given graph) and also investigated the smallest eigenvalue of the Hamming graphs. Let $q \geq 2, d \geq 1$ be integers. Let Q be a set of size q. The Hamming scheme $H(d, q)$ is the association scheme with vertex set Q^d, and as relation the Hamming distance. The $d+1$ relation graphs $H(d, q, j)$, where $0 \leq j \leq d$, have vertex set Q^d, and two vectors of length d are adjacent when they differ in j places. Delsarte showed that the eigenvalues of $H(d, q, j)$ are given by the Krawtchouk polynomials $K_j(i) = \sum_{h=0}^{j}(-1)^h(q-1)^{j-h}\binom{i}{h}\binom{d-i}{j-h}$.

Conjecture 2. [Van Dam and Sotirov 2016] *Let $q \geq 2$ and $j \geq d - \frac{d-1}{q}$ where j is even when $q = 2$. Then the smallest eigenvalue of $H(d, q, j)$ is $K_j(1)$.*

Van Dam and Sotirov verified this conjecture by computer for all pairs (d, q) with $d \leq 30$ and $q \leq 15$. Alon and Sudakov proved the above conjecture for $q = 2, d$ large and j/d fixed. Unbeknownst to these authors, in 2013, Dumer and Kapralova proved the Van Dam-Sotirov conjecture for $q = 2$ and all d.

In this talk, I will describe these connections between the smallest eigenvalue and the max-cut of a graph and our proofs of the Karloff and Van Dam-Sotirov conjectures. Time permitting, I will discuss similar problems for other distance-regular graphs.

Applications of semidefinite programming, symmetry, and algebra to graph partitioning problems

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This is joint work with Renata Sotirov

We will present semidefinite programming (SDP) and eigenvalue bounds for several graph partitioning problems.

The graph partition problem (GPP) is about partitioning the vertex set of a graph into a given number of sets of given sizes such that the total weight of edges joining different sets — the cut — is optimized. We show how to simplify known SDP relaxations for the GPP for graphs with symmetry so that they can be solved fast, using coherent algebras.

We then consider several SDP relaxations for the max-k-cut problem, which is about partitioning the vertex set into k sets (of arbitrary sizes) such that the cut is maximized. For the solution of the weakest SDP relaxation, we use an algebra built from the Laplacian eigenvalue decomposition — the Laplacian algebra — to obtain a closed form expression that includes the largest Laplacian eigenvalue of the graph. This bound is exploited to derive an eigenvalue bound for the chromatic number of a graph. For regular graphs, the new bound on the chromatic number is the same as the well-known Hoffman bound. We demonstrate the quality of the presented bounds for several families of graphs, such as walk-regular graphs, strongly regular graphs, and graphs from the Hamming association scheme.

If time permits, we will also consider the bandwidth problem for graphs. Using symmetry, SDP, and by relating it to the min-cut problem, we obtain best known bounds for the bandwidth of Hamming, Johnson, and Kneser graphs up to 216 vertices.

On mixed Moore-Cayley graphs

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The degree-diameter problem is the problem of finding the largest possible number v of vertices of a graph where both the largest degree k of any vertex and the diameter D of the graph are constrained.

In the undirected case, the corresponding upper bound is given by the *Moore* bound:

$$v \leq 1 + k\frac{(k-1)^D - 1}{k-2}, \tag{1}$$

and a graph attaining this bound is called a *Moore* graph [6]. If $D > 1$, then such a graph is regular of degree k with $k \in \{2, 3, 7, 57\}$, while the existence of a graph corresponding to $k = 57$ is a famous open problem, [1, 4]. In the directed case, where we allow only directed arcs in the graph, a bound similar to Eq. (1) can be derived, but there are no non-trivial graphs attaining this bound, [3, 9].

A graph is said to be *mixed* if it contains both undirected edges and directed arcs. Again, a generalization of the Moore bound in Eq. (1) to the case of mixed graphs can be found, however no mixed graph attaining this bound can exist with diameters greater than 2, [8]. Thus, we may focus on mixed graphs of diameter 2, in which case the Moore bound is given by $v \leq (z+r)^2 + z + 1$, where r is the maximum undirected degree of any vertex and z is the maximum directed out-degree. If equality attains, then every vertex of a mixed Moore graph has the same undirected degree r and directed in/out-degree z, and, moreover, $r = (c^2 + 3)/4$ holds for some odd integer c dividing $(4z - 3)(4z + 5)$, [2]. Unlike the case of undirected Moore graphs, there are an infinite number of feasible pairs (r, z), however, only three mixed Moore graphs with $r > 1$ are known: the Bosák graph on 18 vertices, and the two Jørgensen graphs on 108 vertices, [7]. Furthermore, these three graphs are Cayley graphs and therefore it is interesting to search for more examples of mixed Moore graphs that are also Cayley graphs. With the aid of computer, Erskine [5] ruled out further examples of mixed Moore-Cayley graphs for all orders v up to 485.

In this talk, we give a brief survey on the topic and describe an algebraic approach based on the so-called Higman's method in the theory of association schemes, which enables us to rule out the existence of mixed Moore-Cayley graphs of certain orders.

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Several results on cliques in strongly regular graphs

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This is joint work with Rosemary Bailey, Peter Cameron, Rhys Evans, Alexander Gavrilyuk, Vladislav Kabanov, Dmitry Panasenko, Leonid Shalaginov, Alexandr Valyuzhenich

An equitable t-partition of a graph Γ is a partition of the vertex set of Γ into t parts $P_1, \ldots, P_t$ such that, for all $i, j \in \{1, ..., t\}$, every vertex of P_i is adjacent to the same number, namely, p_{ij}, of vertices of P_j. The matrix $\Pi := (p_{ij})_{i,j=1,\ldots,t}$ is called the quotient matrix of the equitable t-partition.

In [2], a new family of maximal cliques was found in Paley graphs of square order, and it was proved that the new family of cliques comes from an equitable partition. In this talk we discuss this result and its possible generalizations.

It is well known that every eigenvalue of Π is an eigenvalue of the adjacency matrix of Γ. In [1], equitable partitions of Latin-square graphs, whose quotient matrix does not contain the eigenvalue -3, were classified. In this talk we discuss a role of maximal cliques in this classification.

An m-regular clique, in a graph Γ is a clique S such that every vertex of Γ not in S is adjacent to the same positive number m of vertices of S (equivalently, S forms a part of an equitable 2-partition). In the early 1980s, Neumaier [3] studied regular cliques in edge-regular graphs, and a certain class of designs whose point graphs are strongly regular and contain regular cliques. He then posed the problem of whether there exists a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique. In this talk we survey recent results on Neumaier graphs, discuss a determination of the smallest Neumaier graph, which has 16 vertices, and present an infinite family of Neumaier graphs, which extends the smallest one.

Acknowledgments. The work has been supported by RFBR Grant 17-51-560008.

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PC-polynomial on graph and its largest root

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Given a graph G, we are interested on the properties of $\beta(G)$, the largest root of PC-polynomial, a polynomial with integer coefficients depending on the numbers of cliques in G. The number $\beta(G)$ is deeply related to partially commutative algebras, Lovász local lemma and matrices. We find a graph on which $\beta(G)$ reaches the largest value if the numbers $n =$|V| and $k =$|E| are fixed. We find the upper bound on $\beta(G) : \beta(G) < n - (0.941k)/n$ for $n \gg 1$. We obtain new versions of Lovász local lemma. We investigate the analogues of Nordhaus-Gaddum inequalities for $\beta(G)$. Applying random graphs, we prove that the average value of $\beta(G)$ on graphs with n vertices asymptotically equals $\approx 0.672n$.

On directed strongly regular graphs

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A *directed strongly regular graph* (DSRG) with parameters (n, k, t, λ, μ) is a regular directed graph on n vertices with valency k such that every vertex is incident with t undirected edges; the number of directed paths of length 2 directed from a vertex x to another vertex y is λ, if there is an arc from x to y and μ otherwise. Clearly, DSRGs represent a possible generalization of (undirected) strongly regular graphs to the directed case. It was introduced by Duval in [1].

In the talk we present a few constructions of DSRGs.

Firstly, we show a construction based on the Cayley table T of a loop (quasigroup with identity). Let Q be a loop of order n. We refer to the symbol appearing in the x-th row and the y-th column in T as xy. Let us denote the lattice square graph $L_2(n)$, i.e. the graph whose vertices correspond to the cells of the Cayley table T of the loop Q and two vertices are adjacent if and only if they are in the same row or column in T. It is known that the lattice square graph $L_2(n)$ is an undirected strongly regular graph with parameters $(n^2, 2n-2, n-2, 2)$.

By taking two or three copies of $L_2(n)$, respectively, and defining adjacencies between the vertices of different copies, which are based on the operation in the loop Q, we prove the existence of DSRGs with parameter sets $(2n^2, 3n-2, 2n-1, n-1, 3)$, $(2n^2, 4n-2, 2n+2, n+2, 6)$, $(3n^2, 4n-2, 2n, n, 4)$ and $(3n^2, 6n-2, 2n+6, n+6, 10)$ for arbitrary integer n.

Moreover, one of the constructions of DSRGs with parameters $(3n^2, 4n-2, 2n, n, 4)$ seems to be of high interests. It is conjectured that in this case non-isomorphic loops result in non-isomorphic DSRGs. If this conjecture is true, then we have the first evidence of the so-called "prolific construction" of DSRGs, i.e. a construction giving hyper-exponentially many DSRGs with growing n. Similar result is already known in the undirected case of SRGs due to Wallis and Fon-der-Flaass, see [2, 4] and later generalized by Muzychuk [3].

On the other hand it is also conjectured that the (full) group of automorphisms of the DSRG is 6-times larger than the automorphism group of the initial loop. The factor 6 is coming from the permutations of the copies of $L_2(n)$.

Both conjectures were confirmed up to $n \leq 7$ with the aid of a computer.

In the rest of the talk we show constructions of DSRGs which use the technology of the lifts, or voltage assignments.

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Combinatorial curvature for infinite planar graphs

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This is joint works with Yanhui Su (Fuzhou University)

For any planar graph, its ambient space S^2 or R^2 can be endowed with a canonical piecewise flat metric by identifying its faces with regular Euclidean polygons, called the polyhedral surface. The combinatorial curvature of a planar graph is defined as the generalized Gaussian curvature of its polyhedral surface up to the normalization 2π. The total curvature of an infinite planar graph with nonnegative combinatorial curvature will be shown to be an integral multiple of $1/12$ and the number of vertices with non-vanishing curvature is at most 132. Moreover, if the total curvature is positive, then the automorphism group of an infinite planar graph with nonnegative combinatorial curvature is finite.

The Terwilliger algebra of a tree

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Let Γ be a finite connected simple graph. Let X denote the vertex set of Γ and $V = \bigoplus_{x \in X} \mathbb{C}x$ the standard module, i.e., the vector space for which X is an orthonormal basis. Fix a vertex $x_0 \in X$ and let X_i be the set of vertices that have distance i from x_0. Then the standard module V is decomposed into the orthogonal sum $V = \bigoplus_{i=0}^{D} V_i^*$, where $V_i^* = \bigoplus_{x \in X_i} \mathbb{C}x$. The *Terwilliger algebra* $\mathfrak{T}$ of Γ is by definition the subalgebra of $\mathrm{End}(V)$ generated by the adjacency matrix A of Γ and the orthogonal projections $E_i^* : V \to V_i^*$, $0 \leq i \leq D$. Let G be the automorphism group of Γ and H the stabilizer in G of the base vertex x_0: $G = \mathrm{Aut}(\Gamma)$, $H = G_{x_0}$. Then it is easy to see that $\mathfrak{T}$ is contained in the centralizer algebra of H, i.e., each element of $\mathfrak{T}$ commutes with the action of every element of H: $\mathfrak{T} \subseteq \mathrm{Hom}_H(V, V)$.

In this talk, we discuss the Terwilliger algebra of a tree. Precisely speaking, we assume Γ is a rooted tree with x_0 the root and we let $\mathfrak{T}$ be the Terwilliger algebra of Γ with respect to x_0. We show:
(1) $\mathfrak{T} = \mathrm{Hom}_H(V, V)$, i.e., $\mathfrak{T}$ coincides with the centralizer algebra of H.
(2) The $\mathfrak{T}$-module V determines the rooted tree Γ up to isomorphism.
In particular, $\mathfrak{T} = \mathrm{End}(V)$ holds if and only if the rooted tree Γ does not have any symmetry, i.e., $H = 1$.

This talk is based on joint work with Shuang-Dong Li, Jing Xu, Masoud Karimi and Yizheng Fan. We acknowledge that Jack Koolen conjectured: For almost all finite connected simple graphs, $\mathfrak{T} = \mathrm{End}(V)$ holds regardless the base point x_0. This conjecture motivated our study on the Terwilliger algebra of a tree.

Locally Projective Graphs of $GF(2)$-type

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Consider a connected graph Γ with a family $\mathcal{L}$ of complete subgraphs (called lines), and possessing a vertex-transitive automorphism group G preserving $\mathcal{L}$. It is assumed that for every vertex x of Γ there is a $G(x)$-bijection $\pi(x)$ between the set $\mathcal{L}(x)$ of lines containing x of and the point-set of a projective $GF(2)$-space. There is a number of important examples of such *locally projective graphs of* $GF(2)$*-type* where both classical and sporadic simple groups appear among the automorphism groups. The ultimate goal is to classify these graphs up to their local isomorphism. This was achieved by V. I. Trofimov, S. V. Shpectorov and the present author for the case where the lines are of size 2. An approach of extending the classification to the case where the lines are of size 3 will be discussed in the lecture.

Counting colorings of cubic graphs via a generalized Penrose bracket

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A *proper edge coloring* of a cubic graph is a coloring of the edges of the graph using three colors so that three distinct colors appear at each node of the graph. It is well-known that the four-color theorem is equivalent to the statement that every isthmus-free planar cubic graph has at least one proper edge coloring. In [1] Roger Penrose gave a graphical recursion formula, *the Penrose Bracket,* that can be seen to count the number of proper edge colorings of a planar cubic graph. The Penrose Bracket does not count the number of colorings of non-planar cubic graphs. For example, the original Penrose Bracket vanishes on the graph $K_{3,3}$ while this graph has 12 proper edge colorings. In this talk we extend the Penrose Bracket to include any non-planar cubic graph [2] so that the new formula counts the number of proper edge colorings of that graph. The method we use can be explained in the original Penrose context of abstract tensors. We use an immersion into the plane of the (possibly) non-planar graph, and we associate a new tensor to each immersion crossing as well as associating an epsilon tensor to each cubic node of the graph. The result is a new state summation formula that correctly counts the number of colorings of the graph. We will discuss the possible applications of this new Penrose Bracket to map coloring and we will discuss related ways to examine the colorings of cubic graphs. We shall discuss the relationships of this work with knot theory and virtual knot theory [3–5].

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Recent progress on graphs with fixed smallest eigenvalue

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In 1976, Cameron, Goethals, Seidel and Shult showed that any connected graph with smallest eigenvalue at least -2 is a generalized line graph or has at most 36 vertices. One year later, in 1977, Hoffman showed that for $\tau > -1-\sqrt{2}$ any connected graph with smallest eigenvalue at least τ and large enough minimal degree is a generalized line graph and hence its smallest eigenvalue is at least -2. The proof of the result of Cameron et al. uses the classification of the root lattices, whereas Hoffman did not need that. But he had to assume a large minimal degree. In this talk I will explain how to generalize the result of Hoffman to graphs with smallest eigenvalue at least -3 using the natural lattice associated to a graph. On the other hand, I will also show that the result of Cameron et al. is much harder to generalize to graphs with smallest eigenvalue at least -3. This is based on joint work with Akihiro Munemasa (Tohoku University), Masood Ur Rehman (USTC), Qianqian Yang (USTC) and Jaeyoung Yang (Anhui University).

Skew-morphisms and regular Cayley maps for dihedral groups

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This is joint work with Young Soo Kwon

Let G be a finite group having a factorization $G = AB$ into subgroups A and B with B cyclic and $A \cap B = 1$, and let b be a generator of B. The associated skew-morphism is the bijective mapping $f : A \to A$ well defined by the equality $baB = f(a)B$ where $a \in A$. The motivation of studying skew-morphisms comes from topological graph theory. A Cayley map M for a group G is an embedding of a connected Cayley graph Γ over G such that the left translations of G become also map automorphisms. We say that M is regular if its automorphism group $\mathrm{Aut}(M)$ acts regularly on the arcs of Γ. In this case $\mathrm{Aut}(M)$ factorizes as $\mathrm{Aut}(M) = G_\ell Y$, where G_ℓ is the group of all left translations of G and Y is cyclic; in particular, every generator of Y induces a skew-morphism of G. Conversely, knowing all skew-morphisms of G is sufficient to know all regular Cayley maps for G. The term 'Cayley map' appeared first in a paper of Biggs in 1972, and since then Cayley maps have become a well established research topic in algebraic and topological graph theory.

The complete classification of regular Cayey maps for cyclic groups were given by Conder and Tucker [1], and this was the first classification result involving an infinite family of groups. Their approach is group theoretical. Using a result of Conder and Isaacs about the commutator subgroup of a product of an abelian group with a cyclic group, they describe first the possible structure of the automorphism group of the maps, and then sort out those groups which give rise to a map.

In this talk, I discuss regular Cayley maps for dihedral groups. Our approach is partly group theoretical and partly relies on skew-morphism techniques. Let D_n denote the dihedral group of order $2n$ and let C_n be a cyclic subgroup of order n. Using a result of Kovács, Marušič and Muzychuk about groups of order at most $2n^2$ containing D_n and in which C_n is core-free, we classify first the core-free maps. We say that a regular Cayley map M for D_n is core-free if $(C_n)_\ell$ is core-free in $\mathrm{Aut}(M)$. Then, using the fact that an arbitrary map is a cover of a core-free map, we derive several constraints on the skew-morphism induced by the map, which will allow us to find eventually the skew-morphism (and therefore the map as well). The results presented in this talk were obtained together with Young Soo Kwon and can be found in [2–4].

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Delta-matroids and Vassiliev invariants

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Vassiliev (finite type) invariants of knots can be described in terms of weight systems. These are functions on chord diagrams satisfying so-called 4-term relations. There is also a natural way to define 4-term relations for abstract graphs, and graph invariants satisfying these relations produce weight systems: to each chord diagram its intersection graph is associated. The notion of weight system can be extended from chord diagrams, which are orientable embedded graphs with a single vertex, to embedded graphs with arbitrary number of vertices that can well be nonorientable. These embedded graphs are a tool to describe finite order invariants of links: the vertices of a graph are in one-to-one correspondence with the link components. We are going to describe two approaches to constructing analogues of intersection graphs for embedded graphs with arbitrary number of vertices. One approach, due to V. Kleptsyn and E. Smirnov, assigns to an embedded graph a Lagrangian subspace in the relative first homology of a 2-dimensional surface associated to this graph. Another approach, due to S. Lando and V. Zhukov, replaces the embedded graph with the corresponding delta-matroid, as suggested by A. Bouchet in 1980's. In both cases, 4-term relations are written out, and Hopf algebras are constructed. Vyacheslav Zhukov proved recently that the two approaches coincide.

Recent developments in Majorana representations of the symmetric groups

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This is joint work with Clara Franchi and Alexander A. Ivanov

A major difficulty in studying linear representations of certain finite groups, such as the large sporadics, arises when the degrees of these representations become so large that applying the general methods from linear algebra gets hard, if not practically impossible, even by machine computation.

In this talk I'll cope with a problem which is frequent when dealing with the usual representation of the Monster (and many if its simple subgroups) on the Norton-Conway-Griess algebra, or, more generally, with Majorana representations of finite groups, and can be stated as follows: given a finite group G, a complex permutation module V on a finite G-set $\mathcal{X}$, and a G-invariant positive semidefinite hermitian form f, determine the radical $V^{\perp}$ of f from the Gram matrix Γ associated to f with respect to $\mathcal{X}$.

In this context, the G-invariance of the form f implies strong restrictions on the Gram matrix Γ that can be exploited, via the theory of association schemes, to get a significantly more manageable situation. In fact, Γ is equivalent to a block diagonal matrix Γ', whose blocks have sizes corresponding to the multiplicities of the irreducible $\mathbb{C}[G]$-submodules of V, so that the decomposition of $V^{\perp}$ into irreducible $\mathbb{C}[G]$-submodules can be recovered from the ranks of the diagonal blocks of Γ'. The key step to compute the diagonal blocks of Γ' is to determine a generaliseation of the first eigenmatrix of the association scheme related to the action of G on $\mathcal{X}$. If this action is multiplicity-free (or, better, if the graph associated to this action is distance transitive), there are well established combinatorial methods to compute this matrix. On the other hand, if the action is not multiplicity-free, this strategy becomes much more awkward, though still possible in some cases: for example, this machinery has been extended to the case where at most one irreducible $\mathbb{C}[G]$-submodule of the complex permutation module on $\mathcal{X}$ has multiplicity 2 and all the others have multiplicity 1.

I'll describe how, in the case of nontransitive actions (which are definitely not multiplicity-free), $V^{\perp}$ can be determined from the generalised first eigenmatrices of the association schemes related to the actions induced by G on the G-orbits of $\mathcal{X}$.

Around symmetries of vertex operator algebras

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§1. A vertex operator algebra (VOA), introduced by Borcherds in 1986 and formulated by Frenkel–Lepowsky–Meurman in 1988, is a vector space V equipped with a countably many binary operations and elements $\mathbf{1}$ and ω subject to a set of axioms. In particular, the element ω generates an action of the Virasoro algebra, which induces a grading on V. If $\dim V_0 = 0$ and $V_1 = 0$, then $B = V_2$ carries a structure of a commutative nonassociative algebra with an invariant symmetric bilinear form:

$$V = \mathbf{C1} \oplus 0 \oplus B \oplus V_3 \oplus \cdots, \quad \cdot : B \times B \longrightarrow B, \quad (\ |\ ) : B \times B \longrightarrow \mathbf{C}.$$

The one arising from the Moonshine Module $V^\natural$ agrees with the algebra considered by J. Conway in 1985, a variant of the one constructed by R. Griess in 1982 in proving the existence of the Monster, the largest sporadic finite simple group, and the full automorphism group of $V^\natural$ is indeed the Monster. The algebra B of a VOA V with $\dim V_0 = 0$ and $V_1 = 0$ is called the *Griess algebra* of V.

§2. For a VOA V, let V_ω denote the subVOA generated by the element ω. Assume that the following condition is satified for some n:

$$V^{\operatorname{Aut} V} \cap V_\Delta = V_\omega \cap V_\Delta \quad \text{for } \Delta = 0, 1, \ldots, n. \tag{2}$$

When $\dim V_0 = 0$, $V_1 = 0$, and $n = 2m$, the traces $\operatorname{Tr} R_{a_1} \cdots R_{a_m}$ for the adjoint actions of $a_1, \ldots, a_m \in B$ are expressed by the product and the bilinear form (and a totally antisymmetric quintic form for $m = 5$). For $V = V^\natural$, the condition (1) is satisfied for $n = 11$, and the resulting formulae agree with those previously discovered by S. Norton in 1996 in a different way. As the condition (1) for large n imposes severe constraints on the shape of V, such VOAs are seen to be exceptional.

The formulae are proved by using Casimir type elements κ_Δ or the projection $\pi : V \to V_\omega$, for the assumption (1) implies the properties $\kappa_\Delta \in V_\omega$ and $\operatorname{Tr}|_{V_\Delta} o(a) = \operatorname{Tr}|_{V_\Delta} \pi(o(a))$ for $\Delta \leq n$, where $o(a)$ is the action of $a \in V$ that preserves the degree. A VOA satisfying the former property is called an *exceptional VOA* by M. Tuite and the latter a *conformal design* by G. Höhn in a broader context in 2008.

§3. In 1996, M. Miyamoto investigated VOAs with the Griess algebra spanned by vectors e that generate actions of the Virasoro algebra with the central charge $c = 1/2$ of certain type, for which the lowest conformal weights are 0 and 1/2, and found that they induce an action of a 3-transposition group, a group together with a normal subset D of involutions with $|d_i d_j| \leq 3$ for all $d_i, d_j \in D$. The key observation is that the product $e_p \cdot e_q$ of such vectors is either $2e_q$, 0 or a constant multiple of $e_p + e_q - e_{p\circ q}$ in such a VOA, where $e_{p\circ q}$ is determined by e_p and e_q. Now the algebra is seen to be associated with a partial triple system as in the table below (with $\delta = \gamma = 1/2$ in Miyamoto's setting), and the one associated with a Fischer space is related to the corresponding 3-transposition group.

—	$e_p \cdot e_q$	$(e_p \,\vert\, e_q)$
$p = q$	$2e_p$	$\gamma/2$
$p \perp q$	0	0
$p \sim q$	$\delta(e_p + e_q - e_{p\circ q})/2$	$\delta\gamma/2$

By looking at the least eigenvalue of the colinearlity graph of the Fischer space, the 3-transposition groups that act on a VOA as Miyamoto described are classified.

Miyamoto actually studied the cases such as $V^\natural$ as well where B is spanned by vectors e with $c = 1/2$ whose lowest conformal weights are $0, 1/2$ and $1/16$. The VOAs generated by two such vectors e_1, e_2 are classified by S. Sakuma in 2007. A.A. Ivanov then aximatized such algebras in 2009 and called them the *Majorana algebras*. In 2015, J.I. Hall, F. Rehren, and S. Shpectorov introduced and studied a class of algebras called the *axial algebras* as a common generalization of the algebras considered so far here.

Symmetries and combinatorics of finite antilattices

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This is joint work with Karin Cvetko Vah

Antilattices, known also as rectangular quasilattices, form one of the simplest varieties of non-commutative lattices. They are defined as follows: An *anitlattice* is a set N with a pair of binary operations *meet* an *join*, $\wedge$, $\vee$, which are associative, idempotent and satisfy the two absorption laws below:

$$\begin{array}{ll} x \wedge (y \wedge z) = (x \wedge y) \wedge z, & x \vee (y \vee z) = (x \vee y) \vee z, \\ x \wedge x = x, & x \vee x = x, \\ x \wedge y \wedge x = x, & y \vee x \vee y = y. \end{array}$$

In this talk we will explore the combinatorics of finite antilattices via their generating matrices. We will also investigate their substructures, congruences, symmetries, and in particular, their connection with orthogonal latin squares. The inspiration comes from a paper by Jonathan Leech [1]. *Quasiltattices* are defined by the lines above except that the last line is replaced by the formula: $x \wedge y \wedge x = x \iff y \vee x \vee y = y$. A *congruence* on N is equivalence relation that is compatible both with $\vee$ and $\wedge$. A quasilattice is *simple* if it has no non-trivial congruences. The importance of anitlattices lies on the one hand in the Laslo-Leech decomposition theorem [2], which states that a quasilattice is a lattice of antilattices and on the other hand on the fact that a simple quasilattice is either a lattice of an antilattice. Laslo-Leech decomposition theorem is a quasilattice version of the Clifford-McLean theorem for bands, [1,4] We call an antilattice that has no non-trivial sub-algebras *elementary* and antilattices without sub-algberas of even order, *odd*. Among other things, we will describe the relationships between simple, elementary and odd antilattices and the connection of the latter to pairs of orthogonal latin squares. We perform enumeration of various pseudo-varieties of finite antilattices. In particular, we focus our attention to regular antilattices as natural generalization skew antilattices. Recall that a *band* is a semigroup of idempotents. Since each antilattice is obtained as a pair of rectangular bands, we investigate rectangular bands, too and describe some of their properties. When dealing with the symmetry we point out the richness of antilattices. While the automorphism group of a rectangular band must act transitively on the set of its elements, automorphisms of antilattices may give rise to more than one orbit of elements. However, it may happen that an antilattice is transitive. In this respect non-commutative lattices may exhibit more symmetry than ordinary, commutative lattices. In addition to the case of self-dual structures, when the structure is isomorphic to the one in which the operations $\vee$ and $\wedge$ are reversed, the non-commutativity allows us to consider structures that are isomorphic to the ones in which the order of operands is reversed. If time permits we will demonstrate our computer program for working with antilattices.

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Cliques and colourings in GRAPE

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GAP [2] is an internationally developed, freely available, open-source system for algebra and discrete mathematics. GRAPE [6] is a GAP package for computing with finite graphs with associated groups of automorphisms. In GRAPE, a graph Γ always comes together with an associated subgroup A of the automorphism group of Γ (A can be computed by GRAPE or user-specified), and A is used to store Γ efficiently and to speed up computations with Γ.

For many types of combinatorial objects, the construction or classification problem reduces to finding or classifying cliques with certain properties in a problem-specific graph, where often the graph has a large automorphism group. See, for example, [3, 4, 7]. The GRAPE package provides extensive facilities to exploit graph symmetries for clique finding and clique classification (up to the action of the group A of automorphisms associated with the graph). The general functionality in GRAPE for cliques allows for the classification of the cliques with given vertex-weight sum in a vertex-weighted graph (and the weights can often be non-zero d-vectors of non-negative integers), as well as the finding and classification of cliques invariant under a given group of graph automorphisms. This general clique functionality in GRAPE underpins the functionality in the DESIGN package [5] for the discovery and classification of many types of combinatorial designs.

Recently, I have used the clique functionality in GRAPE to develop programs which exploit the automorphism group of a graph Γ to determine, given a positive integer k, whether Γ has a proper vertex k-colouring, and if so, to produce such a colouring. I have used these programs to determine the primitive permutation groups G of degree at most 255, such that G is a group of automorphisms of a non-null non-complete graph having chromatic number equal to its clique number. These groups are precisely the non-synchronizing primitive permutation groups (of degree at most 255), of interest in both permutation group theory and automata theory (see [1]).

I will talk about these recent developments, and give concrete examples and applications of the clique and colouring machinery in GRAPE, so that hopefully you can apply this machinery to your own research problems.

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On 2-closures of primitive solvable permutation groups

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Denote by Ω the set $\{1,\ldots,n\}$, by Sym_n the symmetric group of degree n. Denote the action of Sym_n on Ω^k coordinatewise, i.e. given $\sigma \in Sym_n$ we define

$$\sigma : (x_1,\ldots,x_k) \mapsto (x_1\sigma,\ldots,x_k\sigma).$$

If $G \leq Sym_n$ define the orbits of G on Ω^k by $\Delta_1(k),\ldots,\Delta_m(k)$. Following H.Wielandt we define the k-closure of G (we denote it $G^{(k)}$) by

$$\{\sigma \in Sym_n \mid \Delta_i(k)\sigma = \Delta_i(k) \text{ for } i = 1,\ldots,m\}.$$

In the talk we discuss the possible structure of $G^{(2)}$ for solvable $G \leq Sym_n$.

First we discuss, why we restrict ourself by primitive but not 2-transitive solvable group.

Then we discuss the structure of primitive solvable permutation groups. Namely, if $G \leq Sym_n$ is primitive solvable, then $n = p^k$ for some prime p and $k > 0$. Moreover, G possesses a normal regular elementary abelian subgroup A, and if we denote by L a point stabilizer in G, then $G = A \rtimes L$.

So we may consider L as a subgroup of $GL_k(p)$ acting on the vector space $F_p^k \simeq A$ in a natural way. It is known (see [1, Theorem 2]) that A is a normal subgroup of $G^{(2)}$, so that if K is a point stabilizer of $G^{(2)}$, then $G^{(2)} = A \rtimes K$. Thus one may assume that $L \leq K$. Furthermore, 2-orbits of G are in one-to-one correspondence with of L in its natural action on A. Thus we reduce the original question of finding 2-closure of primitive solvable group to a question of finding 1-closure of irreducible solvable linear groups.

Finally we explain, why we start with primitive linear groups and also give an overview of the results already obtained in this direction.

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Combinatorial games on graphs, Coxeter-Dynkin diagrams, and the geometry of root systems

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Coxeter-Dynkin graphs feature prominently in dozens of topics in modern mathematics, including Lie algebras and Lie groups, reflection groups, regular polytopes, lattice theory, singularities, root systems, von Neumann algebras, quantum groups, knot theory, and many areas of combinatorics.

The ADE graphs and their affine variants are particularly intriguing. Their close connections to symmetries of various kinds has even prompted some physicists to consider E_8 as a unifying object for a theory of everything.

Explaining this remarkable ubiquity of rather simple graphs is a tantalising problem, perhaps first clearly enunciated by Arnold in 1976.

In this talk we consider the graphs as central, and explore two remarkable games, the Numbers game and the Mutation game, that generate quite a lot of associated mathematics around ADE diagrams and their affine variants. The Numbers game was introduced by Moses and also studied by Eriksson. The Mutation game is roughly dual to the Numbers game. Both are played with populations (functions on vertices) on a general graph, and are very simple to define.

We will show that using only elementary analysis of these games already produces a lot of the rich theory that surround these objects. Remarkable lattices and posets will figure, Coxeter/Weyl groups make a natural appearance, the geometry of root systems and connections with representation theory of Lie groups and Lie algebras will figure prominently, and we will also present at least one intriguing challenge.

Character degree graphs of finite groups

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Character degree graphs of finite groups have been investigated intensively in recent years. We will report some new developments.

Contributed talks

Intersection of conjugate solvable subgroups in $GL(n, q)$

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Assume that a finite group G acts on a set Ω. A point $\alpha \in \Omega$ is **G-regular** if $|\alpha^G| = |G|$, i.e. if the stabilizer of α is trivial. Define the action of the group G on Ω^k by

$$g : (\alpha_1, \ldots, \alpha_k) \mapsto (\alpha_1 g, \ldots, \alpha_k g).$$

If G acts faithfully and transitively on Ω then the minimal number k such that the set Ω^k contains a G-regular point is the **base size** of G and is denoted by $b(G)$. For a positive integer m denote the number of G-regular orbits on Ω^m by $Reg(G, m)$ (this number is 0 if $m < b(G)$). If H is a subgroup of G and G acts by the right multiplication on the set Ω of right cosets of H then G/H_G acts faithfully and transitively on Ω. (Here $H_G = \cap_{g\in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

In this work we consider Problem 17.41 b) from the "Kourovka notebook" [2]:

Let H be a solvable subgroup of a finite group G that has no nontrivial solvable normal subgroups. Do there always exist five conjugates of H whose intersection is trivial?

The problem is reduced to the case when G is almost simple in [3]. In particular it sufficient to show that for every solvable subgroup $H < G$

$$Reg_H(G, 5) \geq 5$$

for all almost simple groups G.

In [1] the inequality $Reg_H(G, 5) \geq 5$ is shown for almost simple groups with socle isomorphic to an alternating group A_n, $n \geq 5$.

The main statement of the present work is the following

Theorem. *If S is a maximal solvable subgroup of $G = GL(n, q)$; $q \geq 7$, then $Reg_S(G, 5) \geq 5$.*

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Label classification in machine learning via association schemes

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Machine learning, as an area of computer science, attempts to learn a pattern or labeling system for given set of data, without having the rule for the data programmed ahead of time. We consider the case where some labels are provided for some portion of the data ahead time. For instance, cell phone users often label much of their data and a basic question is to learn how a user will label a new piece of data. We also admit query to a set of points for possible use in a set of training data to be used for learning and additionally allow training examples to be directly used to develop reasoning towards predicting labels of new examples.

More specifically, suppose $X = \{(x_1, L_1), (x_2, L_2), ..., (x_m, L_m)\}$ is a set of labeled data instances with data instances x_i and corresponding labels L_i. We define a non-conformity measure ν_L on X with respect to label L [3]. The idea is that a data instance has high L-non-conformity if it could easily be perceptively labeled with a non-L-label, given isn't "non-conformity" with aspects central to the property of being L. This can be put in the context of a (colored) directed graph: vertex x_i shares an (L_j-colored) edge with vertex x_j provided that $\nu_{L_j}(x_i)$ is below a certain threshold. Now, suppose that a new data point x_{m+1} from the query set is considered for possible inclusion in training data. In the parlance of statistics, suppose that a null hypothesis assumes that x_{m+1} has class label L. Following [1], we define a p-value function P_L by

$$P_L(x_{m+1}) = \frac{count\{k : \nu_L(x_k) \geq \nu_L(x_{m+1})\}}{card(X)}$$

We determine whether x_{m+1} should be selected to re-train a classifier by considering a p-value closeness matrix C whose entries $C_{ij} = |P_i - P_j|$ contain the differences of all p-values for x_{m+1} according to class labels i, j. An appeal to the Perron-Frobenius Theorem ensures that a unique maximum positive eigenvalue exists. This is used to determine a data-driven, rather than an empirically-driven, threshold for selection. Furthermore, we are able to frame results in the context of association schemes [2].

Acknowledgments. We are grateful to Akihiro Munemasa for sharing the work contained in [2].

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The properties of the set of subarcs of a symmetric irrational dendrite

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Let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be a system of contracting similarities in $\mathbb{R}^2$.
A non-empty compact $K \subset \mathbb{R}^2$ is called *the attractor of the system* $\mathcal{S}$, if $K = \bigcup\limits_{i=1}^{m} S_i(K)$.
The system $\mathcal{S}$ is called *postcritically finite* (PCF), if the set $\{x \in K : \exists i_1 \ldots i_n, j, l : S_{i_1 \ldots i_n}(x) \in K_j \cap K_l\}$ is finite.

It was proved by C. Bandt [1], that K is the attractor of a postcritically finite system $\mathcal{S}$, then the set of dimensions of shortest subarcs $\gamma \subset K$ is finite.

The properties of non-PCF self-similar dendrites are still unexplored.

In the present paper we consider a generalisation of the construction of polygonal dendrites from [2]. We construct a system of four mappings $\mathcal{S} = \{S_1, S_2, S_3, S_4\}$ of an equilateral triangle with vertices $(0; 0), (1; 0)$ and $(1/2; \sqrt{3}/2)$ to itself, and prove the following proposition.

Proposition 1.
(i) *The system $\mathcal{S}$ is not postcritically finite.*
(ii) *The attractor K of the system $\mathcal{S}$ is a self-similar dendrite.*
(iii) *All subarcs γ in K have the same Hausdorff dimension α.*
(iv) *The set of α-dimensional Hausdorff measures of the arcs γ_{Ox} with endpoints $O = (1/2, \sqrt{3}/6)$ and $x \in [0, 1] \cap K$ is a self-similar Cantor discontinuum.*

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On edge-transitive factorizations of complete uniform hypergraphs

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This is joint work with Zaiping Lu

As a generalization of vertex-transitive self-complementray graph, homogeneous factorization of complete graph have been an active topic in recent years. We generalize the transitive factorizations of graphs to transitive factorizations of complete uniform hypergraphs. we classify the edge-transitive homogeneous factorizations of $\mathcal{K}_n^k$ $(k \geq 3)$ and the symmetric factorizations of $\mathcal{K}_n^k$ $(k \geq 3)$. Based on some classical results on simple groups and Kantor's classification of k-homogeneous but not k-transitive permutation groups (1972), we give several algebraic constructions for edge-transitive homogeneous factorizations of $\mathcal{K}_n^k$ $(k \geq 3)$ and symmetric factorizations of $\mathcal{K}_n^k$ $(k \geq 3)$. As a corollary of our classification result, we obtain all the symmetric self-complementary k-uniform hypergraphs, extending Peisert's classification of symmetric self-complementary graphs in 2001. Among these symmetric factorizations of $\mathcal{K}_n^k$ with $k \geq 3$, only 8 of them are not 1-factorizations. As for edge-transitive homogeneous factorizations of $\mathcal{K}_n^k$ with $k \geq 3$, no 1-factorization arises.

On groups whose proper Schur rings are commutative

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A Schur ring $\mathcal{A}$ is called *Dedekind* if the formal sum of every $\mathcal{A}$-subgroup is in the center of $\mathcal{A}$. In this talk, we find all finite groups G such that every proper Schur ring over G is Dedekind.

Theorem 1. *Every proper Schur ring over G is Dedekind if and only if G is a Dedekind group or $G \cong D_n$ where $n = 4$ or n is a Fermat prime.*

As a consequence of this theorem, we find all finite groups G such that every proper Schur ring over G is commutative or symmetric, respectively.

Corollay 2. *Every proper Schur ring over G is commutative if and only if G is an abelian group, $G \cong Q_8$ or $G \cong D_n$ where $n = 4$ or n is a Fermat prime.*

Corollary 3. *Every proper Schur ring over G is symmetric if and only if G is an elementary abelian 2-group or $G \cong C_n$ where $n = 4$ or n is a Fermat prime.*

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Kernels in 3-regular circulant digraphs

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We are interested in the following open problem in [1]: characterize circulant digraphs which have kernels.

Recall that a kernel J in a digraph D is an independent set of vertices of D such that for every vertex $w \in V(D) \setminus J$, there is an arc from w to a vertex in J.

Subtraction game (also called take-away game) is a well-known impartial combinatorial game (cf. [2], [3], [4], [5]). We use the theory of three elements subtraction game to study the existence of kernels in some 3-regular circulant digraphs, which are directed analogy of circulant graphs (cf. [6]).

The following is the rule of the game. There are a pile of n coins and a set $S = \{a, b, c\}$ of three positive integers, where $a < b < c$. Two players move alternately, subtracting a, b or c from the number of coins. The player who make the last move wins.

Definition (cf. [7]) *A circulant digraph $D = D_n(a, b, c)$ on n vertices with three pairwise distinct jumps a, b, c has vertices $i - a, i - b, i - c \pmod n$ adjacent to each vertex i.*

The main result of the talk is as follows.

Theorem *Suppose that a, b, c are positive integers such that $b \in (a, 2a)$, and $c = m(a + b) + t$, where $m \in \{0, 1, 2, \cdots\}$.*

(1) If m is positive and $t \in [b - a, a)$, then there is a kernel in the circulant digraph $D = D_{b+c}(a, b, c)$.

(2) If $t \in [a, b]$, then there is a kernel in the circulant digraph $D = D_{a+b}(a, b, c)$.

(3) If $t \in (b, a + b]$, then there is a kernel in the circulant digraph $D = D_{a+c}(a, b, c)$.

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Deformations of polygonal dendrites in the plane

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Let $P\subset\mathbb{R}^2$ be a finite polygon homeomorphic to a disk, $V_P = \{A_1, ..., A_{n_P}\}$ be set of its vertices. We consider such a system of similarities $\mathcal{S} = \{S_1, \dots, S_m\}$ in $\mathbb{R}^2$ that:
(D1) for any $i \in I$ set $P_i = S_i(P)\subset P$;
(D2) for any $i \neq j$, $i, j \in I$, $P_i \cap P_j = V_{P_i} \cap V_{P_j}$;
(D3) $V_P\subset \bigcup\limits_{i\in I} S_i(V_P)$;
(D4) the set $\widetilde{P} = \bigcup\limits_{i=1}^{m} P_i$ is contractible.

The system $\mathcal{S}$, satisfying the conditions (**D1–D4**), is called a *contractible P-polygonal system of similarities.* As it was proved in [1], the attractor K of such system $\mathcal{S}$ is a dendrite.
If $\mathcal{S}$ satisfies **(D2–D4)** only, it is called a *generalised polygonal system.*

A generalised polygonal system $\mathcal{S}'$ is called δ-deformation of a P-polygonal system $\mathcal{S}$, if there is such homeomorphism $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, that $|f(x) - x| < \delta$ for any $x \in \mathbb{R}^2$ and $f(P_k) = P'_k$ for any $k = 1, ..., m$.

A vertex $A \subset V_P$ is called a *cyclic vertex*, if there is such multiindex $\mathbf{j} = j_1 j_2 \dots j_k$, that $S_\mathbf{j}(A) = A$. If $S_\mathbf{j}(z) = re^{i\varphi}(z - A) + A$, then the *parameter λ_A of the cyclic vertex A* is the number $\dfrac{\varphi}{\log r}$, where the choice of the value of φ is defined by the geometric configuration of the system $\mathcal{S}$.

A point $B \in \cup_{i=1}^{m} V_{P_i}$ is *subordinate* to a cyclic vertex A, if for some multiindex $\mathbf{j}, S_\mathbf{j}(A) = B$.

Parameter matching condition. *We say that a system $\mathcal{S}$ satisfies the parameter matching condition if for any $B \in \cup_{i=1}^{m} V_{P_i}$ and cyclic vertices A, A' such B is subordinate to both A and A', $\lambda_A = \lambda_{A'}$.*

If the parameter matching condition for the generalised polygonal system eS is violated at some point $B \in \cup_{i=1}^{m} V_{P_i}$, then its attractor K is not simply-connected near the point B and therefore is not a dendrite.

Theorem. *For any contractible P-polygonal systen $\mathcal{S}$ there is such $\delta > 0$, that for any δ-deformation $\mathcal{S}'$ of the system $\mathcal{S}$, satisfying the parameter matching condition, the attractor $K(\mathcal{S}')$ is a dendrite, homeomorphic to $K(\mathcal{S})$.*

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Construction of fullerenes and Pogorelov polytopes

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This is joint work with Victor Buchstaber

A Pogorelov polytope (Pog-polytope) is a combinatorial simple convex 3-polytope that has a bounded right-angled realization in the Lobachevsky (hyperbolic) space $\mathbb{L}^3$. Theorems by A. V. Pogorelov (1967) and E. M. Andreev (1970) imply that such polytopes are characterised by the condition that they are different from the 3-simplex and have no 3- and 4-belts, where a k-belt is a cyclic sequence of facets with empty common intersection such that facets are adjacent if and only if they follow each other. This condition appeared in the work by G. D. Birkhoff (1913), who proved that the 4 Colors Conjecture could be proved only for this class of polytopes and even for a smaller class with an additional restriction that any 5-belt surrounds a facet (Pog^*-polytopes). In graph theory graphs of Pog and Pog^*-polytopes are known as planar cyclically and strongly cyclically 5-edge-connected ($c5$- and c^*5-connected). In 1974 D. Barnette and J. W. Butler proved that any $c5$-connected planar graph is obtained from the graph of the dodecahedron by a sequence of operations of 3 types: an addition of an edge (A_1), a subdivision of a pentagon (A_2), and an addition of a pair of edges (A_3). A k-barrel B_k is a 3-polytope with the surface glued from two disks consisting of a k-gon surrounded by 5-gons. B_k is obtained from B_{k-1} by A_3. In 1977 D. Barnette proved that any c^*5-connected planar graph is obtained from the graph of some B_k, $k \geq 5$, by a sequence of operations A_1, and any $c5$-connected planar graph is obtained from some B_k, $k \geq 5$, by a sequence of operations A_1 and A_2. There is a special case of A_1 when we cut off two adjacent edges of a polytope by one plane (a $(2,k)$-truncation, where k is the number of edges of a face spanned by the two edges). Combining methods by D. Barnette and the authors we obtain.

Theorem 1. [2, 3] *A simple 3-polytope P is Pog if and only if either $P = B_k$, $k \geq 5$, or P is obtained from B_5 or B_6 by a sequence of $(2,k)$-truncations, $k \geq 6$, and operations A_2. It is a Pog^*-polytope if and only if either $P = B_k$, $k \geq 5$, or P is obtained from B_6 by a sequence of $(2,k)$-truncations, $k \geq 6$.*

Results by T. Dǒslić (1998, 2003) imply that any fullerene (a simple 3-polytope with only 5- and 6-gonal faces) is a Pog-polytope. Results by F. Kardoš, F. Škrekovski and K. Kutnar, D. Marušič (2008) imply that a fullerene is not Pog^* if and only if it is a $(5,0)$-nanotube (i.e. is obtained from B_5 by a sequence of A_3 applied to a 5-gon surrounded by 5-gons). Denote by $\mathcal{F}_{5,\leq 7}$ the set of all simple 3-polytopes with 5-, 6- and at most one 7-gon, where the 7-gon is adjacent to a 5-gon. These polytopes are Pog ([2]).

Theorem 2. [2] *Any fullerene different from a $(5,0)$-nanotube can be obtained from B_6 by a sequence of $(2,6)$ and $(2,7)$-truncations such that intermediate polytopes belong to $\mathcal{F}_{5,\leq 7}$.*

Theorem 3. [3] *A polytope in $\mathcal{F}_{5,\leq 7}$ is not Pog^* iff $P \neq B_5$ and P contains a 5-gon surrounded by 5-gons. Such a polytope is obtained from a fullerene by a sequence of A_2. Any other polytope $P \in \mathcal{F}_{5,\leq 7}$ is obtained from B_6 by a sequence of $(2,6)$- and $(2,7)$-truncations, and operations O_1,O_2,O_3, where O_i are certain compositions of these truncations, such that intermediate polytopes also belong to $\mathcal{F}_{5,\leq 7}$.*

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Neumaier graphs

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This is joint work with Sergey Goryainov and Dmitry Panasenko

A regular clique, or more specifically an m-regular clique, in a graph Gamma is a clique S such that every vertex of Gamma not in S is adjacent to the same positive number m of vertices of S. In the early 1980s, Neumaier [1] studied regular cliques in edge-regular graphs, and a certain class of designs whose point graphs are strongly regular and contain regular cliques. He then posed the problem of whether there exists a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique.

The first known examples of Neumaier graphs appear in Goryainov and Shalaginov [2], each of which has 24 vertices. In [3], Greaves and Koolen construct an infinite family of Neumaier graphs having 1-regular cliques. Indeed, before our work, all known Neumaier graphs had at least 24 vertices, and contained m-regular cliques only for the value $m = 1$.

In this talk, I will first focus on our determination of the smallest Neumaier graph, which has 16 vertices, valency 9 and a 2-regular 4-clique. Then, I will present a new infinite sequence of Neumaier graphs, the i^{th} element of which contains a 2^i-regular clique.

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The study of energy states in the Thomson problem

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One of the most important contemporary problems experimental physicists face is calculation of coordinates of an optimal nanostructure. An example is the problem of coating of carbon nanotubes by a suitable fullerene. A nontrivial character of the problem is provided by an exponentially increasing number of suitable halves of the molecule among which an optimal configuration must be chosen guided by the stability of the chemical bond between carbon atoms [1].

The well-known Thomson problem has received unexpected development in the context of this problem. It was noted that through the transition to the so-called dual lattice, the problem of finding a suitable fullerene is reduced to solving the problem similar to the Thomson problem on a hemispherical surface [2].

The main difficulty in solving the problem is not the search for optimal solutions, but their classification. The generally accepted approach is that the configurations are considered equivalent if their potential energies coincide with the specified accuracy [3, 4]. The correct way of classification is to check overlaying solutions using rotations and reflections.

The task was set: to develop a method that allows us to classify the solutions of the Thomson problem depending on the position of point charges on a sphere. To solve the problem, a complete weighted graph was constructed. As vertices of the graph are chosen point charges. In this case, the compatibility of configurations reduces to the isomorphism problem for complete weighted graphs.

The result of the study is a method that allows to undertake research of the solutions of the Thomson problem. In addition, an auxiliary software product was written that allows to quickly generate Thomson problem solutions, as well as to check the emerging solutions for compatibility.

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On triple intersection numbers of association schemes

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This is joint work with Janoš Vidali

Let $\mathfrak{X} = (X, \{R_0, \ldots, R_D\})$ be a symmetric association scheme of D classes. For a 3-tuple xyz of points of X, let $[\ell, m, n] := [\ell, m, n]_{x,y,z}$ denote the *triple intersection number* (with respect to xyz) defined by:

$$[\ell, m, n] = \#\{w \in X \mid wR_\ell x, wR_m y, wR_n z\}.$$

Unlike the intersection numbers, the triple intersection numbers $[\ell, m, n]$ depend, in general, on the choice of x, y, z. On the other hand, vanishing of some of the Krein parameters of $\mathfrak{X}$ often leads to non-trivial linear Diophantine equations involving triple intersection numbers as the unknowns (perhaps, it was first observed by Cameron, Goethals and Seidel in [2], see also [1, Theorem 2.3.2]). This fact has been used to compute triple intersection numbers of certain putative distance-regular graphs with feasible intersection arrays, from which non-existence of the corresponding graphs has been shown, see [3–5]. An implementation of this approach is now available as a part of a package for the Sage computer algebra system for checking feasibility of a given intersection array of a distance-regular graph, [6].

Recently, Williford [7] has published lists of feasible Krein parameters for primitive 3-class Q-polynomial association schemes on up to 2800 vertices, and for Q-bipartite (but not Q-antipodal) 4- and 5-class association schemes on up to 10000 and 50000 vertices, respectively. In this work, by computing triple intersection numbers, we rule out many open cases from these lists. If time permits, we will discuss a generalization of triple intersection numbers to quadruples.

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Graphs and metrics: the partition case

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Given an ordered partition $\Pi = \{P_1, P_2, ..., P_t\}$ of the vertices of a connected graph G, the *partition representation* of a vertex $v \in V$ with respect to the partition Π is the vector $r(v|\Pi) = (d(v, P_1), d(v, P_2), ..., d(v, P_t))$, where $d(v, P_i)$, with $1 \leq i \leq t$, represents the distance between the vertex v and the set P_i, that is $d(v, P_i) = \min_{u \in P_i}\{d(v, u)\}$. The partition Π is a *resolving partition* of G if for every pair of distinct vertices $u, v \in V$, $r(u|\Pi) \neq r(v|\Pi)$ (in such case, we also say that there is a set $P_i \in \Pi$ which resolves the pair of vertices u, v). The *partition dimension* of G is the minimum number of sets in any resolving partition of G and is denoted by $pd(G)$. Concepts above were presented first in [1].

On the other hand, a set W of vertices of G *strongly resolves* two different vertices $x, y \notin W$, if either $d_G(x, W) = d_G(x, y) + d_G(y, W)$ or $d_G(y, W) = d_G(y, x) + d_G(x, W)$. An ordered vertex partition $\Pi = \{U_1, U_2, ..., U_k\}$ of a graph G is a *strong resolving partition* for G if every two different vertices of G belonging to the same set of the partition are strongly resolved by some set of Π. A strong resolving partition of minimum cardinality is called a *strong partition basis* and its cardinality the *strong partition dimension*, which is denoted by $pd_s(G)$. Concepts above were presented first in [2].

Now, the Cartesian product of two graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$ is the graph $G \Box H = (V, E)$, such that $V = \{(a, b) \ : \ a \in V_1, \ b \in V_2\}$ and two vertices $(a, b), (c, d) \in V$ are adjacent in $G \Box H$ if and only if, either

- $a = c$ and $bd \in E_2$, or
- $b = d$ and $ac \in E_1$.

Also,the strong product of G and H is the graph $G \boxtimes H = (V, E)$, such that $V = \{(a, b) \ : \ a \in V_1, \ b \in V_2\}$ and two vertices $(a, b), (c, d) \in V$ are adjacent in $G \boxtimes H$ if and only if, either

- $a = c$ and $bd \in E_2$, or
- $b = d$ and $ac \in E_1$, or
- $ac \in E_1$ and $bd \in E_2$.

For more information on product graphs we recommend the book [3]. Several results concerning the (strong) partition dimension of graphs with some emphasis in product graphs are presented in this work.

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On a connection between the order of a finite group and the set of conjugacy classes size

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In this paper, all groups are finite. The number of elements of a set π is denoted by $|\pi|$. Denote the set of prime divisors of positive integer n by $\pi(n)$, and the set $\pi(|G|)$ for a group G by $\pi(G)$. The greatest power of a prime p dividing the natural number n will be denoted by n_p. For a set of prime π and a natural number n we will denote $n_\pi = \prod_{p\in\pi} |n|_p$.

Let G be a group and take $a \in G$. We denote by a^G the conjugacy class of G containing a. Put $N(G) = \{|x^G|, x \in G\} \setminus \{1\}$. Denote by the $|G||_p$ number p^n such that $N(G)$ contains α multiple of p^n and avoids the multiple of p^{n+1}. For $\pi \subseteq \pi(G)$ put $|G||_\pi = \prod_{p\in\pi} |G||_p$. For brevity, write $|G||$ to mean $|G||_{\pi(G)}$. Observe that $|G||_p$ divides $|G|_p$ for each $p \in \pi(G)$. However, $|G||_p$ can be less than $|G|_p$.

Definition. *Let p and q be distinct numbers. Say that a group G satisfies the condition $\{p,q\}^*$ and write $G \in \{p,q\}^*$ if we have $\alpha_{\{p,q\}} \in \{|G||_p, |G||_q, |G||_{\{p,q\}}\}$ for every $\alpha \in N(G)$.*

A. R Camina (see [1]) proved that a group G with $\{p,q\}^*$-property is nilpotent if $N(G) = \{1, p^n, q^m, p^n q^m\}$. A. Beltram and M.J. Filipe (see [2])extended Camina's theorem in the following way: let G be a finite soluble group whose conjugacy class sizes are $\{1, n, m, nm\}$, where n and m are coprime positive integers; then G is nilpotent and the integers n and m are prime-power numbers. Q. Kong and X. Guo (see [3]) investigated groups such that the set of conjugacy class sizes of biprimary elements is precisely $1, p^\alpha, m, p^\alpha m$, where p^α is a prime power, $(p,m) = 1$ and there is a p-element whose conjugacy class size is p^α. They proved that in this case such groups is nilpotent and $m = q^\beta$ for some prime number $q \neq p$.

In the general case, a group with the $\{p,q\}^*$-property is not nilpotent. For example, let $G \simeq L_n(k)$. Then $G \in \{p,q\}$, where p is a primitive prime divisor of $k^n - 1$ and q is a primitive prime divisor of $k^{n-1} - 1$.

In this paper we inspect the groups with $\{p,q\}^*$-properties and trivial center.

Theorem. *If $G \in \{p,q\}^*$ is a group with trivial center, where $p,q \in \pi(G)$ and $p > q > 5$, then $|G|_{\{p,q\}} = |G||_{\{p,q\}}$.*

Corollary. *In the hypotheses of the theorem, $C_G(g) \cap C_G(h) = 1$ for every p-element g and every q-element h.*

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Coincidence of Gruenberg–Kegel graphs of non-isomorphic finite groups

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This is joint work with Natalia Maslova

Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e. the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel graph* (or the *prime graph*) $\Gamma(G)$ of G; in this simple graph the vertex set is $\pi(G)$, and distinct vertices p and q are adjacent if and only if $pq \in \omega(G)$. Denote the number of connected components of $\Gamma(G)$ by $s(G)$, and the set of connected components of $\Gamma(G)$ by $\{\pi_i(G) \mid 1 \leq i \leq s(G)\}$; for a group G of even order we assume that $2 \in \pi_1(G)$. The following problem is of interest.

Problem. *Describe cases when Gruenberg–Kegel graphs of non-isomorphic finite groups coincide.*

Firstly, in the present talk we give a survey of some existing results.

Secondary, we concentrate on the following partial case.

Recall, a finite group G is a *Frobenius group* if there is a non-trivial subgroup C of G such that $C \cap gCg^{-1} = \{1\}$ whenever $g \notin C$. A subgroup C is a *Frobenius complement* of G. Let

$$K = \{1\} \cup (G \setminus \bigcup_{g \in G} gCg^{-1}).$$

Then K is a normal subgroup of a Frobenius group G with a Frobenius complement C. Note that K is called the *Frobenius core* of G.

Recall, the *socle* $Soc(G)$ of a finite group G is the subgroup of G generated by the set of all its non-trivial minimal normal subgroups. A finite group G is *almost simple* if $Soc(G)$ is a finite nonabelian simple group. It is well-known that a finite group G is almost simple if and only if there exists a finite nonabelian simple group S such that $S \cong Inn(S) \trianglelefteq G \leq \mathrm{Aut}(S)$.

Gruenberg–Kegel Theorem. *If G is a finite group with disconnected Gruenberg–Kegel graph, then one of the following statements holds*:

(1) *G is a Frobenius group*;

(2) *G is a 2-Frobenius group, i. e., $G = ABC$, where A and AB are normal subgroups of G, AB and BC are Frobenius groups with cores A and B and complements B and C, respectively*;

(3) *G is an extension of a nilpotent $\pi_1(G)$-group by a group A, where $S \trianglelefteq A \leq \mathrm{Aut}(S)$, S is a finite nonabelian simple group with $s(G) \leq s(S)$, and A/S is a $\pi_1(G)$-group.*

The cases when Gruenberg–Kegel graphs of a finite nonabelian simple group and of a group from item (1) or (2) of the Gruenberg-Kegel theorem coincide were described in [1]. The cases when Gruenberg–Kegel graphs of a finite almost simple group and of a solvable group from item (1) or a group from item (2) of the Gruenberg-Kegel theorem coincide can be found with using of [2].

In the present talk we consider the cases when Gruenberg–Kegel graphs of a finite almost simple group and of a nonsolvable group from item (1) of the Gruenberg-Kegel theorem coincide.

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Second eigenmatrices of a non-commutative association schemes obtained from steiner systems

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Definition 1. Let X be a finite set of order n and $R_0, R_1, \dots, R_d \subset X \times X$. The *adjacency matrix* A_i is a $(0,1)$-matrix indexed by X such that $(A_i)_{x,y} = 1$ if $(x,y) \in R_i$ and $(A_i)_{x,y} = 0$ otherwise. Let I_n, J_n be the identity matrix and all-ones matrix of order n, respectively. The pair $(X, \{R_i\}_{i=0}^d)$ is called an *association scheme of class* d if the following hold:

(i) $A_0 = I_n$.

(ii) $\sum_{i=0}^d A_i = J_n$.

(iii) For any $i \in \{0,1,\dots,d\}$, there exists $i' \in \{0,1,\dots,d\}$ such that $A_{i'} = A_i^T$.

(iv) For any $i,j,k \in \{0,1,\dots,d\}$, there exists $p_{i,j}^k$ such that $A_iA_j = \sum_{k=0^d} p_{i,j}^k A_k$.

An association scheme is commutative if $A_iA_j = A_jA_i$ holds for any $i,j \in \{0,1,\dots,d\}$ and non-commutative otherwise. The algebra spanned by $A_0, A_1, \dots, A_d$ over $\mathbb{C}$ is called a *adjacency algebra.* Since the adjacency algebra is semisimple, it is isomorphic to $\bigoplus_{k=1}^r M_{d_k}(\mathbb{C})$ for uniquely determined positive integers $r, d_1, d_2, \dots, d_r$, where $\sum_{k=0}^r d_k^2 = n$. We may construct a set $\{E_k^{(i,j)} \mid 1 \le k \le r, 1 \le i,j \le d_k\}$ as a basis (see [3]). Since the adjacency algebra has $\{A_0, A_1, \dots, A_d\}$ as a basis, there exist $q_k^{(i,j)}(l)$ such that

$$E_k^{(i,j)} = \frac{1}{n}\sum_{l=0}^d q_k^{(i,j)}(l)A_l.$$

The square matrix $Q = (q_k^{(i,j)}(l))$ is called a second eigenmatrix.

Let V be a finite set of order v and $\mathcal{B}$ be a set of k-subsets of V. The pair $(V, \mathcal{B})$ is called a t-(v,k,λ)design if $\lambda = \#\{B \in \mathcal{B} \mid S \subset B\}$ holds for any t-subset S of V. Let $\mathcal{F} = \{(p,V) \in V \times \mathcal{B} \mid p \in B\}$. In particular, t-$(v,k,1)$-design is called a steiner system. In [2], for steiner systems with $t = 2$, $(\mathcal{F}, \{R_i\}_{i=0}^6)$ is a non-commutative association scheme of class 6.

A quotient association scheme of class 2 is constructed from the non-commutative association scheme $(\mathcal{F}, \{R_i\}_{i=0}^6)$. The quotient association scheme is commutative, and the primitive idempotents of the quotient association scheme correspond to central idempotents of $(\mathcal{F}, \{R_i\}_{i=0}^6)$. In my talk, we reveal the correspondence and construct the second eigenmatrix of $(\mathcal{F}, \{R_i\}_{i=0}^6)$.

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Prolific construction of strictly Deza graphs

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A non-empty k-regular graph Γ on n vertices is called a *Deza graph* if there exist constants b and a such that any pair of distinct vertices of Γ has either b or a common neighbours. We assume further that $b \geq a$. The quantities n, k, b, and a are called the parameters of Γ and are written as the quadruple (n, k, b, a).

The concept of Deza graphs was introduced in 1999 by M. Erickson, S. Fernando, W. Haemers, D. Hardy, and J. Hemmeter in the seminal paper [1] influenced by A. Deza and M. Deza [2]. Deza graphs generalize strongly regular graphs in the sense that the number of common neighbours of any pair of vertices in a Deza graph does not depend on their adjacency.

A strongly regular graph has diameter 2, except for the trivial case of a disjoint union of complete graphs. Unlike the strongly regular graphs, Deza graphs can have diameter greater than 2. If a Deza graph has diameter 2 and is not strongly regular, then it is called *a strictly Deza graph.*

In [1] a basic theory of strictly Deza graphs was developed and several ways to construct such graphs were introduced. Some other links to bibliography on Deza graphs could be found in the homepage of M. Deza [3].

W.D. Wallis proposed in [5] a new construction of strongly regular graphs based on an affine design and a Steiner 2-design. Later D.G. Fon-Der-Flaass found how to introduce a sort of randomness into Wallis construction. In [4] he built hyperexponentially many strongly regular graphs with the same parameters.

We show how to modify W.D. Wallis and Fon-Der-Flaass ideas in order to get a new prolific constructions of strictly Deza graphs.

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On the semigroup of similarities with unique one point intersection, not satisfying WSP

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Let $\mathcal{S} = \{S_1, \ldots, S_m\}$ be a system of contraction similarities in $\mathbb{R}^n$. The system $\mathcal{S}$ is said to satisfy the *open set condition* (OSC), iff there exists an open set O such that $S_i(O) \subset O$ and $S_i(O) \cap S_j(O) = \varnothing$ for all distinct $i, j \in I = \{1, \ldots, m\}$.

Denote by $F = \{S_{\mathbf{i}} : \mathbf{i} \in I^\infty\}$ the semigroup, generated by $\mathcal{S}$; then $\mathcal{F} = F^{-1} \circ F$, or a set of all compositions $S_{\mathbf{j}}^{-1} S_{\mathbf{i}}$, $\mathbf{i}, \mathbf{j} \in I^*$, is the *associated family of similarities*. The system $\mathcal{S}$ has the *weak separation property* (WSP) iff $\mathrm{Id} \notin \overline{\mathcal{F} \setminus \mathrm{Id}}$. If the system doesn't have WSP, then it doesn't satisfy OSC, but the opposite is not true.

A nonempty compact set $K = K(\mathcal{S})$ such that $K = \bigcup\limits_{i=1}^{m} S_i(K)$, is called an *attractor* of the system $\mathcal{S}$, or a *self-similar set*. A set $C(\mathcal{S}) = \bigcup\limits_{i=1, j \neq i}^{m} S_i(K) \cap S_j(K)$ is called a *critical set* of the system $\mathcal{S}$.

The violation of OSC is caused by overlaps of an attractor of system $\mathcal{S}$. If OSC does not hold, there is at least one point in C, but there is no guarantee that this point is unique - no such examples were constructed before.

Known methods like transversality method, which was used in [1], allow to construct the systems $\mathcal{S}_p$, depending of parameter p, such that $\mathcal{S}_p$ does not satisfy WSP (so as OSC) for Lebesgue-almost all p; but using this method we cannot control the type of overlaps.

Our method, based on General Position Theorem [2, Theorem 14], allows us to construct a families of self-similar sets with predictable behavior of critical set. For example, in [2] we get exact overlap for double fixed points. In the current work we prove the existence of system with unique one point intersection, not satisfying WSP.

We define a system $\mathcal{S}_{pq} = \{S_1, S_2, S_3, S_4\}$ of contraction similarities on $[0, 1]$ by the equations $S_1(x) = px$, $S_2(x) = h(q) - qx$, $S_3(x) = h(q) - \dfrac{1-x}{16}$, $S_4(x) = 1 - \dfrac{1-x}{16}$, where the contraction ratios $p, q \in (0, 1/16)$, and $h(q) = \dfrac{1+q}{1+16q}$. Let K_{pq} be the *attractor* of the system $\mathcal{S}_{pq}$.

From the construction it follows that $S_i(K_{pq}) \cap S_j(K_{pq}) \neq \varnothing$ with $i \neq j$ if and only if $\{i, j\} = \{3, 4\}$. We prove the following:

Theorem 1. *For Lebesgue-almost all* $(p, q) \in (0, 1/16)^2$*:*
(1) $C(S_{pq})$ *consists of one point* $h(q)$*;*
(2) $\mathcal{S}_{pq}$ *do not satisfy WSP.*

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Non-residually finite direct limits of the 2-Bridge link groups

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This is joint work with Donghi Lee

A group G is said to be *residually finite* if for each $1 \neq g \in G$ there is a finite group H and a homomorphism $\varphi : G \longrightarrow H$ such that $\varphi(g) \neq 1$. Most of well-known finitely presented groups are residually finite, and constructions of non-residually finite groups are rare.

One of the representative open problem is whether every hyperbolic group is residually finite, and it is commonly belived that a non-residually finite hyperbolic group exists.

The relative hyperbolicity and the small cancellation conditions of finitely presented groups are closely related to hyperbolic groups. So we construct a non-Hopfian group (i.e. non-residually finite) satisfying the small cancellation conditions $C(4)$ and $T(4)$ by taking a direct limit of relatively hyperbolic groups $\{G_n \,|\, n = 0, 1, 2, \cdots\}$. Each G_n has the form $G_n = \langle a, b \,|\, u_{r_0} = u_{r_1} = u_{r_2} = \cdots = u_{r_n} = 1\rangle$, where u_{r_i} is the relator of the presentation of the 2-bridge link group of slope r_i for each $i \geq 0$. There are specific rational numbers which satisfy the above conditions.

Theorem 1. *Let $r_i = [7, i\langle 3, 1, 1\rangle, 4, 5]$ for every integer $i \geq 0$. Also for every integer $n \geq 0$, let $G_n = \langle a, b \,|\, u_{r_0} = u_{r_1} = \cdots = u_{r_n} = 1\rangle$ which is hyperbolic relative to a set $\{H_n, K_n\}$ of groups. Here, $H_n = \langle a, v_{r_0}, v_{r_1}, \ldots, v_{r_n}\rangle$ and $K_n = \langle b, w_{r_0}, w_{r_1}, \ldots, w_{r_n}\rangle$ are proper subgroups of G_n, where $(u_{r_i}) \equiv (av_{r_i}a^{-1}v_{r_i}^{-1}) \equiv (w_{r_i}bw_{r_i}^{-1}b^{-1})$ for every $i = 0, 1, \ldots, n$. Then the direct limit G of a sequence*

$$G_0 \twoheadrightarrow G_1 \twoheadrightarrow G_2 \twoheadrightarrow \cdots$$

equipped with the canonical epimorphism $\alpha_n : G_n \twoheadrightarrow G_{n+1}$ at each $n \geq 0$ is infinitely presented as $G = \langle a, b \,|\, u_{r_0} = u_{r_1} = u_{r_2} = \cdots = 1\rangle$ which satisfies small cancellation conditions $C(4)$ and $T(4)$, and is non-Hopfian.

The symbol "$i\langle 3, 1, 1\rangle$"represents i successive $(3, 1, 1)$'s. Let M be a sufficiently large integer. Then we can obtain the group $\mathfrak{G}_n := \langle a, b \,|\, a^M = b^M = u_{r_0} = u_{r_1} = \cdots = u_{r_n} = 1\rangle$ is hyperbolic for each integer $n \geq 0$.

Theorem 2. *Let $r_i = [7, i\langle 3, 1, 1\rangle, 4, 5]$ for every integer $i \geq 0$. Also for every integer $n \geq 0$, let $\mathfrak{G}_n = \langle a, b \,|\, a^M = b^M = u_{r_0} = u_{r_1} = \cdots = u_{r_n} = 1\rangle$ which is hyperbolic. Then the direct limit G of a sequence*

$$\mathfrak{G}_0 \twoheadrightarrow \mathfrak{G}_1 \twoheadrightarrow \mathfrak{G}_2 \twoheadrightarrow \cdots$$

equipped with the canonical epimorphism $\alpha_n : \mathfrak{G}_n \twoheadrightarrow \mathfrak{G}_{n+1}$ at each $n \geq 0$ is infinitely presented as $\mathfrak{G} = \langle a, b \,|\, a^M = b^M = u_{r_0} = u_{r_1} = u_{r_2} = \cdots = 1\rangle$, and is non-Hopfian.

Until Theorem 2, we could construct non-residually finite $\mathfrak{G}$ such that $\mathfrak{G}$ is still infinitely presented. In order to construct a hyperbolic group, we need to have the group finitely presented at least. In the near future (hopefully before I give a talk on G2R2 conference), we expect to construct another group by adding some generators or relators which results in deleting some relators so that the group becomes finitely presented.

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Classification of t-balanced regular Cayley maps on some groups

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In this talk, we will consider t-balanced regular Cayley maps on semidirect product of $\mathbb{Z}_n$ and $\mathbb{Z}_2$. It is well-known that any semidirect product of $\mathbb{Z}_n$ and $\mathbb{Z}_2$ is isomorphic to the group $\langle a, b \mid a^n = b^2 = 1,\ ba = a^r b\rangle$ for some r satisfying $r^2 \equiv 1(\text{mod } n)$. We denote this group by $\Gamma(n, r)$. If $r = 1$ and -1, then $\Gamma(n, r)$ is isomorphic to $\mathbb{Z}_n \times \mathbb{Z}_2$ and dibedral group of order $2n$, respectively. For a positive integer $n = 4m$ divided by 4, if $r = 2m - 1$, then $\Gamma(n, r)$ is called semi-dihedral group. For dihedral groups and semi-dihedral groups, t-balanced regular Cayley maps on these groups were classified in [1] and [2]. In this talk, for arbitrary r satisfying $r^2 \equiv 1(\text{mod } n)$, we consider the classification of t-balanced regular Cayley maps on $\Gamma(n, r)$.

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On the spectra of Cayley graphs

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Let G be a group with identity 1_G, and let A, B be two disjoint subsets such that $1_G \notin A \cup B$ and such that $A = A^{-1}$. The Cayley mixed graph $\mathrm{Cay}(G; A, B)$ consists of the set of vertices given by the elements of G, the edges join $g \in G$ and gx for every $x \in A$, and there is an arc from vertex $g \in G$ towards gx for every $x \in B$. Hence $\mathrm{Cay}(G; A, B)$ is totally regular of undirected degree $|A|$ and directed degree $|B|$. In this talk, we present the spectra of Cayley graph $\mathrm{Cay}(G; A, B)$ on some group G and discuss cospectral nonequivalent such Cayley graphs.

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On perfect 2-colorings of infinite multipath graphs

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This is joint work with Olga Parshina

A coloring of vertices of a graph G with two colors (black and white) is called *perfect coloring* with parameter matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, if every white vertex has exactly a white neighbors and b black ones, parameters c and d are defined analogously for black vertices.

Consider *an infinite path graph* C_∞, whose set of vertices is the set of integers, and two vertices are adjacent, if they are on the distance 1.

Let G be a transitive graph. We put a copy of graph G into each vertex of the graph C_∞, add edges between every two vertices from neighboring copies. We call this graph *an infinite G-times path* and denote $C_\infty \cdot G$. The graph defined above is exactly the lexicographical product of the graph C_∞ and the graph G. Let n be a positive integer. The subject of the research is perfect 2-colorings of $\overline{K_n}$- and K_n-times paths.

Graphs under consideration have extensive structure, in other words they contain the graph C_∞ as a subgraph. Circulant graphs are similar to them in this sense. Perfect colorings of these graphs are considered in several papers (e.g. see [1]).

Any coloring of the graph $C_\infty \cdot \overline{K_n}$ and $C_\infty \cdot K_n$ is periodic. Denote the period of such a coloring by the $2 \times l$ table, where l is the number of $\overline{K_n}$- or K_n-blocks in this period. Components of first string correspond to numbers of white vertices in such blocks, and components of second string – to numbers of black ones.

Let x, y, z and t be positive integers, that are less than n. Perfect colorings of the $\overline{K_n}$-times path with periods $\begin{pmatrix} x & y & z & t \\ n-x & n-y & n-z & n-t \end{pmatrix}$ under the condition $x + z = y + t$ are called *standard. Standard* perfect colorings of the graph $C_\infty \cdot K_n$ are defined as colorings with periods $\begin{pmatrix} x & y & z \\ n-x & n-y & n-z \end{pmatrix}$.

Perfect colorings of these graphs are described in following theorems.

Theorem 1. *Up to renaming of colors, the perfect 2-colorings of the graph $C_\infty \cdot \overline{K_n}$ are exhausted by the standard perfect colorings and two sporadic ones:*

$$\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}, \quad \begin{pmatrix} n & 0 & 0 \\ 0 & n & n \end{pmatrix}.$$

Theorem 2. *Up to renaming of colors, the perfect 2-colorings of the graph $C_\infty \cdot K_n$ are exhausted by the standard perfect colorings and two sporadic ones:*

$$\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}, \quad \begin{pmatrix} n & n & 0 & 0 \\ 0 & 0 & n & n \end{pmatrix}.$$

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Integral Cayley graphs over finite groups

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Suppose that S is a nonempty subset of a finite group G, containing with every element its inverse, i. e. $S = S^{-1} = \{s^{-1} \mid s \in S\}$. The *Cayley graph* $\Gamma = Cay(G,S)$ of a group G associated with S is an undirected graph with the vertex set identified with G, and vertices $g, h \in G$ are joined by an edge if and only if there exists $s \in S$ such that $s = g^{-1}h$. A graph Γ is said to be *integral*, if all eigenvalues of its adjacency matrix are integers [1].

Integral Cayley graphs over abelian, dihedral and cyclic groups were investigated in [2–4].

In this talk we present new results on integral Cayley graphs over finite groups.

Theorem 1. *Let G be a finite nilpotent group and $S = S^{-1}$ be a nonempty subset of G. If S is normal, i. e. $S^G = S$, and with every element $s \in S$ it contains also all generators of the cyclic group $\langle s\rangle$, then $\Gamma = Cay(G,S)$ is integral.*

Corollary. *A Cayley graph $\Gamma = Cay(G,S)$ of a 2-group G generated by a normal set S of involutions is integral.*

Theorem 2. *Let $G = S_n$ be the symmetric group of degree $n \geqslant 2$ and S be the set of all transpositions of G. Then the graph $\Gamma = Cay(G,S)$ is integral.*

Theorem 3. *Let $G = A_n$ be the alternating group of degree $n \geqslant 2$ and $S = \{(1ij) \mid 2 \leqslant i,j \leqslant n, i \neq j\}$. Then the graph $\Gamma = Cay(G,S)$ is integral. Its spectrum coincides with the set*

$$\{-n+1, 1-n+1, 2^2-n+1, \ldots, (n-1)^2-n+1\}.$$

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Polarities of projective planes and related (hyper)graphs

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This is joint work with Francesco Pavese and Leo Storme

Polarity graphs were first introduced by Erdős and Rényi [1] to solve a problem in extremal graph theory. Ever since, they have appeared in several contexts, most often in extremal (hyper)graph theory. While their applications are very fruitful, this interesting class of graphs merits an investigation of its own. As a motivating example, a natural question that any mathematician can understand is the following.

Question. Consider the finite vector space $V(3, q)$. What is the largest set of vectors A such that no two distinct vectors in A are orthogonal?

This particular question is related to the independence number of certain polarity graphs. The techniques that have been employed to investigate this problem range from algebraic graph theory and coherent configurations [2], to purely geometrical constructions. In this talk, I will give an overview of the current best known lower and upper bounds on the independence numbers of certain polarity (hyper)graphs, including some new results, which are joint work with Francesco Pavese and Leo Storme [3, 4].

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Arithmetics and combinatorics of circulant graphs

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We study arithmetical and combinatorial properties of a few infinite families of circulant graphs. Among them are circulant graphs with constant jumps, circulant graphs with odd valency of vertices and circulant graphs with unbounded jumps. We investigate the Smith normal form of Laplacian for these graphs and determine the structure of the corresponding critical group [1]. The obtained results are applied to find the number of spanning trees for the above mentioned families of graphs [2].

Exact analytical formulas for the number of spanning trees in terms of Chebyshev polynomials are derived, and their asymptotics is found. As a consequence, we show that the thermodynamic limit of a family of circulant graphs coincides with the small Mahler measure of the associated Laurent polynomial. Also, we investigate pure arithmetical properties for the number of spanning trees of circulant graphs.

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Jacobian and complexity of the I-graphs

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We consider a family of I-graphs $I(n, k, l)$, which is a generalization of the class of generalized Petersen graphs [1]. In the present paper, we develop a new method for counting Jacobian group [2, 3] and apply it for the I-graph $I(n, k, l)$. We show that the minimum number of generators of $Jac(I(n, k, l))$ is at least two and at most $2k + 2l - 1$. Also, we obtain a closed formula for the number of spanning trees of $I(n, k, l)$ in terms of Chebyshev polynomials. We investigate some arithmetical properties of this number and its asymptotic behaviour.

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On regular subgroups of the automorphism group of the Hamming code of length 15

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Given a binary vector $x \in F_2^n$ and permutation $\pi \in S_n$ we consider the transformation (x, π), that sends vector $y \in F_2^n$ to $x+\pi(y)$. *The automorphism group* $Aut(F_2^n)$ of F_2^n (bijections preserving Hamming metric) is the set of all such transformations w.t.r composition: $(x, \pi) \cdot (y, \pi') = (x + \pi(y), \pi \circ \pi')$.

The setwise stabilizer of a binary code C of length n in $Aut(F_2^n)$ is called the *automorphism group* $Aut(C)$ of C. If there is a subgroup H of $Aut(C)$ acting transitively and $|H| = |C|$, then H acting on C is called a *regular group* and C is called a *propelinear code* [1].

Obviously, any linear code is propelinear. The Hamming code is known to have the largest order of the automorphism group in the class of perfect binary codes of any fixed length [3] and supposed to have maximum number of regular subgroups of its automorphism group among propelinear perfect codes. However, the fact that the order of the automorphism group of the Hamming code of length n is $|GL(log(n + 1), 2)|2^{n-log(n+1)}$ makes attempts of complete classification of regular subgroups impossible for ordinary calculational machinery for smallest nontrivial length 15.

We say that a group $H, H < Aut(F_2^n)$ *is narrow-sense embedded* in a subgroup $G, G < Aut(F_2^n)$, if $H < G$ and $\{\pi : (x, \pi) \in H\} = \{\pi : (x, \pi) \in G\}$.

We considered Nordstrom-Robinson and Hadamard codes that are known to be subcodes of the Hamming code of length 15. These codes have automorphism groups of small order, relatively to that of the Hamming code of length 15 and the classification of regular subgroups of the automorphism groups of both codes was obtained using MAGMA.

Theorem 1. *There are 73 and 39 conjugacy classes of regular subgroups of the automorphism group of Nordstrom-Robinson and Hadamard codes of length 15 respectively, that fall into 45 and 11 isomorphism classes respectively.*

The automorphism groups of Nordstrom-Robinson and Hadamard codes are subgroups that of the Hamming code of length 15, which argues for narrow-sense embedding of regular subgroups.

Theorem 2. *1. The regular subgroups of the Nordstrom-Robinson code are narrow-sense embedded in 605 conjugacy classes of regular subgroups of the automorphism group of the Hamming code of length 15, which fall into at least 219 isomorphism classes. 2. The regular subgroups of the Hadamard code of length 15 are narrow-sense embedded in at least 1207 congugacy classes of regular subgroups of the automorphism group of the Hamming code of length 15, which fall into at least 48 isomorphism classes.*

Remark. We see that there are at least 219 nonisomorphic regular subgroups of the Hamming code of length 15, which significantly exceeds the known lower bounds for the number of nonisomorphic regular subgroups of the automorphism groups of other propelinear perfect codes of length 15, see [2].

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On equitable 2-partitions of Hamming graphs $H(n,q)$ with eigenvalues λ_2

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A r-partition $C_1,\dots,C_r$ of the vertex set of a graph is called *equitable* with quotient matrix $A=(A_{kl})_{k,l\in\{1,\dots,r\}}$ if for any $k,l\in\{1,\dots,r\}$ a vertex from C_k has A_{kl} neighbors in C_l. We treat equitable partitions as functions on the vertex sets, which take value k on vertices from C_k for $k\in\{1,\dots,r\}$.

An eigenvalue of an equitable partition is an eigenvalue of its matrix of parameters. Any eigenvalue of an equitable partition of a graph is an eigenvalue of the graph, which is known as Lloyd theorem [2].

The Hamming graph $H(n,q)$ is the direct product of n copies of the complete graph K_q. The eigenvalues of $H(n,q)$ are $\lambda_i(n)=(q-1)n-qi$, $i\in\{0,\dots,n\}$. Let the function $f':V(H(n',q))\to\{1,\dots,r\}$ be obtained from an equitable r-partition f of $H(n,q)$, $n<n'$ by adding nonessential coordinates: $f'(x_1,\dots,x_{n'})=f(x_1,\dots,x_n)$. The function f' is an equitable partition of $H(n',q)$ [3]. Moreover, $\lambda_i(n')$ is an eigenvalue of f' iff $\lambda_i(n)$ is an eigenvalue of f. An equitable partition of $H(n,q)$ is called *reduced* if every its coordinate is essential.

The following characterization was obtained in [1]: the only reduced equitable 2-partitions of the Hamming graph $H(n,q)$ with eigenvalue $\lambda_1(n)$ are those with $n=1$. Therefore the consideration of those with $\lambda_2(n)$ is naturally to be addressed.

Construction A. Let $q=2t$ and consider a partition of $V(K_q)$ into complete graphs with vertex sets V_1 and V_2, $|V_1|=|V_2|=t$. Let C_1 be the following subset of vertices $H(4,q)$:

$$(V_1\times V_1\times V_1\cup V_1\times V_1\times V_2\cup V_2\times V_1\times V_2\cup V_2\times V_1\times V_1)\times V_1\cup$$

$$(V_2\times V_2\times V_1\cup V_2\times V_2\times V_2\cup V_2\times V_1\times V_2\cup V_2\times V_1\times V_1)\times V_2.$$

Proposition 1 [4]. *$C_1,\overline{C_1}$ is a reduced equitable 2-partition of $H(4,q)$ with eigenvalue $\lambda_2(n)$.*

Construction B (Permutation switching construction). Consider a partition of $V(K_q)$ into $n-1$ complete graphs with vertex sets $V_1,\dots,V_{n-1}$. For any $i\in\{1,\dots,n-1\}$ let $f_i:V_i\times V(K_q)\to\{1,2\}$ be an equitable 2-partition with eigenvalue -2 such that $\sum_{\alpha\in V(K_q)}f_i(x_1,\alpha)=c$, where c is independent on the choices of i in $\{1,\dots,n\}$ and x_1 in V_i. Define the function $f:V(H(n,q))\to\{1,2\}$ as follows: for any $i\in\{1,\dots,n-1\}$ if x_1 in V_i then for all $x_j\in V(K_q)$, $j\geq 3$ we have $f(x_1,x_2,\dots,x_n)=f_i(x_1,x_2)$. The function f is an equitable 2-partition of $H(n,q)$ with eigenvalue $\lambda_2(n)$. Further, the construction allows permutation switchings to be applied. Let π_i be the transposition $(2,i+1)$. Define $\widehat{f}:V(H(n,q))\to\{1,2\}$ to be such that for any $i\in\{1,\dots,n-1\}$, x_1 in V_i $\widehat{f}(x_1,\dots,x_n)=f_i(\pi_i(x_1,\dots,x_n))$.

Proposition 2. *The function $\widehat{f}$ is a reduced equitable 2-partition of $H(n,q)$ with the eigenvalue $\lambda_2(n)$.*

Theorem. *The only reduced equitable 2-partitions of $H(n,q)$ with eigenvalue $\lambda_2(n)$ are either reduced equitable partitions of $H(2,q)$ or $H(3,q)$ or the partitions obtained by constructions A or B.*

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Non-reductive homogeneous spaces, admitting affine connections

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The purpose of the work is a study of three-dimensional non-reductive homogeneous spaces, admitting invariant affine connections, description of the affine connections together with their curvature and torsion tensors, holonomy algebras.

Let $(\overline{G}, M)$ be a three-dimensional homogeneous space, where $\overline{G}$ is a Lie group on the manifold M. We fix an arbitrary point $o \in M$ and denote by $G = \overline{G}_o$ the stationary subgroup of o. We can correspond the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ of Lie algebras to $(\overline{G}, M)$, where $\bar{\mathfrak{g}}$ is the Lie algebra of $\overline{G}$ and $\mathfrak{g}$ is the subalgebra of $\bar{\mathfrak{g}}$ corresponding to the subgroup G. This pair uniquely determines the local structure of $(\overline{G}, M)$, two homogeneous spaces are locally isomorphic if and only if the corresponding pairs of Lie algebras are equivalent. An *isotropic* $\mathfrak{g}$*-module* $\mathfrak{m}$ is the $\mathfrak{g}$-module $\bar{\mathfrak{g}}/\mathfrak{g}$ such that $x.(y+\mathfrak{g}) = [x, y]+\mathfrak{g}$. The corresponding representation $\lambda\colon \mathfrak{g} \to \mathfrak{gl}(\mathfrak{m})$ is called an *isotropic representation* of $(\bar{\mathfrak{g}}, \mathfrak{g})$. The pair $(\bar{\mathfrak{g}}, \mathfrak{g})$ is said to be *isotropy-faithful* if its isotropic representation is injective.

We classify (up to isomorphism) faithful three-dimensional $\mathfrak{g}$-modules U. This is equivalent to classifying all subalgebras of $\mathfrak{gl}(3, \mathbb{R})$ viewed up to conjugation. For each obtained $\mathfrak{g}$-module U we classify (up to equivalence) all pairs $(\bar{\mathfrak{g}}, \mathfrak{g})$ such that the $\mathfrak{g}$-modules $\bar{\mathfrak{g}}/\mathfrak{g}$ and U are isomorphic.

Invariant affine connections on $(\overline{G}, M)$ are in one-to-one correspondence with linear mappings $\Lambda\colon \bar{\mathfrak{g}} \to \mathfrak{gl}(\mathfrak{m})$ such that $\Lambda|_{\mathfrak{g}} = \lambda$ and Λ is $\mathfrak{g}$-invariant. We call this mappings *(invariant) affine connections* on the pair $(\bar{\mathfrak{g}}, \mathfrak{g})$. If there exists at least one invariant connection on $(\bar{\mathfrak{g}}, \mathfrak{g})$ then this pair is isotropy-faithful [1].

It appears that the isotropy-faithfulness is not sufficient for the pair in order to have invariant connections. We say that a homogeneous space $\overline{G}/G$ is *reductive* if the Lie algebra $\bar{\mathfrak{g}}$ may be decomposed into a vector space direct sum of the Lie algebra $\mathfrak{g}$ and an $\mathrm{ad}(G)$-invariant subspace $\mathfrak{m}$, that is, if $\bar{\mathfrak{g}} = \mathfrak{g} + \mathfrak{m}$, $\mathfrak{g} \cap \mathfrak{m} = 0$ and $\mathrm{ad}(G)\mathfrak{m} \subset \mathfrak{m}$. Last condition implies $[\mathfrak{g}, \mathfrak{m}] \subset \mathfrak{m}$ and, conversely, if G is connected. If a homogeneous space is reductive, then the space always admits an invariant connection.

We describe all three-dimensional non-reductive homogeneous spaces, allowing invariant affine connections (the local classification of such spaces is equivalent to the description of the effective pairs of Lie algebras) and all invariant affine connections on the spaces together with their curvature and torsion tensors, holonomy algebras.

Studies are based on the use of properties of the Lie algebras, Lie groups and homogeneous spaces and they mainly have local character. The peculiarity of techniques presented in the work is the application of purely algebraic approach for description of homogeneous spaces and connections on them, as well as combination of methods of differential geometry, the theory of Lie groups and algebras and the theory of homogeneous spaces.

The results of the work can be used to study of the manifolds, and have applications in various fields of mathematics and physics, since many fundamental problems in these areas is associated with study of invariant objects on homogeneous space. The methods presented in the work can be used for the analysis of physical models, and algorithms can be computerized and used for the decision of similar problems in large dimensions.

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Compact n-manifolds via $(n+1)$-colored graphs

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In this work we extend the representation of compact 3-manifolds by 4-colored graphs given in [1], and developed in [2] and [3], to the general dimension $n \geq 3$. In this way, any compact PL n-manifold with (possibly empty) boundary without spherical components can be represented via $(n+1)$-colored graphs (i.e. regular $(n+1)$-valent graphs endowed by a proper edge-coloration with $n+1$ colors). This type of representation was introduced by Pezzana's school in Modena in the seventies, but only for the closed case.

Any $(n+1)$-colored graph Γ induces an n-dimensional quasi-manifold $\widehat{M}_\Gamma$ with singular set S_Γ of dimension $\leq n-3$. By removing from $\widehat{M}_\Gamma$ the interior of the regular neighborhood of S_Γ, we obtain an n-dimensional manifold M_Γ with a boundary without spherical components. Using this construction, any PL compact n-manifold with non-empty non-spherical boundary can be (non-uniquely) represented by $(n+1)$-colored graphs, as in the closed case.

In this context we prove the following result.

Theorem 1. *Let Γ be an $(n+1)$-colored graph and let $\Sigma_c(\Gamma)$ be the $(n+2)$-colored graph obtained from Γ by doubling all edges of a fixed color c, then $\widehat{M}_{\Sigma_c(\Gamma)} = \Sigma(\widehat{M}_\Gamma)$, where $\Sigma(\cdot)$ denotes the suspension of the space $\cdot$. Moreover, if M_Γ is not a sphere, then $M_{\Sigma_c(\Gamma)} = M_\Gamma \times [0,1]$.*

As a consequence, we obtain a graph representation Γ_m of the $(m+n)$-manifold $M^n \times B^m$ (where B^m is the m-ball), for any $m \geq 1$, starting from a graph representation Γ of an n-manifold $M^n \neq S^n$, just by adding m parallel edges for any edge of a fixed color of Γ. It is worth noting that the graphs Γ_m and Γ have the same order.

Dipole moves connecting different graphs representing the same manifold, graph techniques for the computation of the fundamental groups of the represented spaces and (boundary-) connected sums of graphs inducing (boundary-) connected sums of the represented manifolds are introduced and/or discussed.

All these constructions and results are included in [4].

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On equitable partitions of divisible design graphs

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An equitable t-partition of a graph Γ is a partition of the vertex set of Γ into t parts $P_1, \dots, P_t$ such that, for all $i, j \in \{1, ..., t\}$, every vertex of P_i is adjacent to the same number, namely, p_{ij}, of vertices of P_j. The matrix $\Pi := (p_{ij})_{i,j=1,\dots,t}$ is called the quotient matrix of the equitable t-partition. It is well known that every eigenvalue of Π is an eigenvalue of the adjacency matrix of Γ.

A k-regular graph Γ with v vertices is called a *divisible design graph* with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ if the vertex set can be partitioned into m classes of size n, such that two distinct vertices from the same class have exactly λ_1 common neighbors, and two vertices from different classes have exactly λ_2 common neighbors (see [1]). By [1, Lemma 2.1], a divisible design graph has at most five eigenvalues, which can be expressed in terms of the parameters.

In this work we study and classify equitable 2-partitions for several families of divisible design graphs.

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Perfect 2-colorings of infinite circulant graphs with a continuous set of odd distances

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Consider an infinite graph $Ci_\infty(d_1, d_2, d_3, ..., d_n)$, whose set of vertices is the set of integers, and two vertices are adjacent if they are on the distance $d \in \{d_1, d_2, d_3, ..., d_n\}$. Let us call this construction an *infinite circulant graph.* A finite graph $Ci_t(d_1, d_2, d_3, ..., d_n)$ is a graph with the set of vertices coinciding with the set Z_t, for every vertex v having the multiset of incident edges $\{(v, v + d_i \text{ mod t})|i = 1, 2, ..., n\}$. There is a natural homomorphism from the set of vertices of the graph $Ci_\infty(d_1, d_2, d_3, ..., d_n)$ on the set of vertices of $Ci_t(d_1, d_2, d_3, ..., d_n)$ corresponding to the homomorphism from Z to Z_t.

Let k be a positive integer. A k-*coloring* of vertices of a graph $G = (V, E)$ is a map $\varphi : V \to \{1, 2, ..., k\}$. The value $\varphi(v)$ for a vertex $v \in V$ is called a *color* of v. A k-coloring of vertices is called *perfect*, if for every pair (i, j), where i, j are not necessarily distinct integers from the set $\{1, 2, \ldots, k\}$ there is an uniquely defined non-negative integer α_{ij} equals to the number of vertices of the color j in the neighborhood of each vertex of the color i. The *period* T of a coloring is a sequence $[\gamma_1\gamma_2...\gamma_t]$, where $\gamma_i = \varphi(v_{m+i})$ for an integer m, and $\varphi(v_l) = \varphi(v_{l+jt})$ for any choice of integers l and j. The coloring of a regular graph is uniquely defined by its period.

Perfect 2-colorings of circulant graphs are considered in [1, 2]. We are interested in so-called circulant graphs with a continuous set of odd distances, i.e. in ones with the property $d_i = 2i - 1, i = 1, 2, 3, ..., n$. Let us note, that for every positive integer n the graph $Ci_\infty(1, 3, 5, \ldots, 2n - 1)$ is regular of degree $2n$ and is bipartite.

We state the following conjecture.

Conjecture. *Let k and n be positive integers. The set of perfect k-colorings of a graph $Ci_\infty(1, 3, 5, \ldots, 2n - 1)$ consists of all perfect k-colorings of graphs $Ci_t(1, 3, 5, \ldots, 2n - 1)$ for $t = 4n - 2, 4n, 4n + 2$ and the following four:* $[123...k]$, $[123...(k-1)k(k-1)...32]$, $[123...(k-1)kk(k-1)...32]$, $[123...(k-1)kk(k-1)...321]$.

We are going to present the proof of this conjecture in case of $k = 2$ and for arbitrary n. A similar conjecture for infinite circulants $Ci_\infty(1, 2, \ldots, n), n \in \mathbb{N}$ was posed in [3]. It was proved in case $k = 2$ and $n \in \mathbb{N}$ in [1], and in case $n = 2$ for arbitrary k in [4]. The natural homomorphism from n-dimensional grid Z^n on $Ci_\infty(1, 2, \ldots, n)$ shows that the problem of complete description of perfect colorings in arbitrary number of colors of such graphs is rather complicated.

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Construction of pairs of orthogonal latin cubes based on combinatorial designs

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A *latin square* of order n is an $n \times n$ array of n symbols in which each symbol occurs exactly once in every row and in every column. A 3-dimensional array satisfying the same condition is called a *latin cube*. Two latin squares are *orthogonal* if, when they are superimposed, every ordered pair of symbols appears exactly once. Two latin cubes are orthogonal if every pair of latin squares corresponding to 2-dimensional faces of cubes are orthogonal.

Let Q be a set of cardinality n. A subset C of Q^d is called an *MDS code* (of order n with code distance $t+1$ and with length d) if $|C \cap \Gamma| = 1$ for each t-dimensional face Γ. It is well known that pairs of orthogonal latin squares are equivalent to MDS codes with code distance 3 and length 4, and pairs of orthogonal latin cubes are equivalent to MDS codes with code distance 3 and length 5.

For any prime power n, $n \geq 4$, there exist linear MDS codes of order n with code distance 3 and length 5. Consequently, there exist pairs of orthogonal latin cubes of order n. Construction based on Cartesian product provides pairs of orthogonal latin cubes of order $n_1 n_2$ if pairs of orthogonal latin cubes of orders n_1 and n_2 exist. In [4] Wilson's type construction ([1]) was used to obtain pairs of orthogonal latin cubes of order $n = 16(6k \pm 1) + 4$. Then it remains to consider orders $2k_1$ and $3k_2$ (in part) for which $\gcd(k_1, 2) = 1$ and $\gcd(k_2, 3) = 1$.

A *Steiner system* with parameters τ, d, n, $\tau \leq d$, written $S(\tau, d, n)$, is a set of d-element subsets of Q (called *blocks*) with the property that each τ-element subset of Q is contained in exactly one block. Recently Keevash [2,3] showed that the natural divisibility conditions are sufficient for existence of Steiner system apart from a finite number of exceptional n given fixed τ and d. Moreover, he proved that for any $S(\tau, d, n)$ design there exists $S(\tau+1, d, n)$ design that contains the first design if n is sufficiently large and arithmetic conditions hold.

In [5] MDS codes are used to construct Steiner quadruple systems $S(3, 4, n)$. Now we present a construction of orthogonal latin cubes based on combinatorial designs.

Proposition. *If designs D_1 of type $S(2,5,n)$ and D_2 of type $S(3,5,n)$ exist and $D_1 \subset D_2$, then there exits a pair of orthogonal latin cubes of order n.*

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A simple multibody system on a discrete circle

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This is joint work with Yaokun Wu

Let n be a positive integer and let $\mathbb{Z}_n$ denote the cyclic group $\mathbb{Z}/n\mathbb{Z}$ of residue classes of integers modulo n, and let the integers $i \in \mathbb{Z}$ also denote their residue classes $i + n\mathbb{Z} \in \mathbb{Z}_n$ as long as no confusion can result. For each subset A of $\mathbb{Z}_n$, we call $i \in \mathbb{Z}_n$ a head of A provided $i \in A$ and $i-1 \notin A$, and we call $i \in \mathbb{Z}_n$ a tail of A provided $i \in A$ and $i+1 \notin A$. We call a subset of $\mathbb{Z}_n$ a proper interval if it has a unique head i and a unique tail j and we designate this proper interval by $[i,j]_n$. We often use $[j]_n$ for $[1,j]_n$.

We play a roulette game on a discrete circle with n slots. The n slots placed circularly can be naturally identified with $\mathbb{Z}_n$. We choose a positive integer k satisfying $2 \leq k \leq n$ and put k undistinguished balls on k slots of the circle. Suppose that the slots of the k balls are read as $a_1, \dots, a_k \in \mathbb{Z}_n$ such that the interval with head a_i and tail a_{i+1} contains exactly two balls, one at slot a_i and one at slot a_{i+1}, for $i = 1, \dots, k-1$. In one step movement, the ball at slot a_i can move to any slot from the interval $[a_i, a_{i+1}-1]_n$ with equal probability. This gives an ergodic Markov chain with $\binom{n}{k}$ states. The support of this Markov chain is an Eulerian digraph $\mathcal{R}_{n,k}$ of diameter k and with $\mathbb{Z}_n$ as its automorphism group. The probability of visiting any state in the stationary distribution of the Markov chain is proportional to the degree of the state in this Eulerian digraph.

We study the linear algebra around the above simple k-body problem on a discrete circle. We use $R_{n,k}$ for the linear map from $\mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$ to itself that sends $A \in \binom{\mathbb{Z}_n}{k}$ to $\sum B$ where B runs through all out-neighbors of A in the digraph $\mathcal{R}_{n,k}$; we also use $R_{n,k}$ for the matrix of the corresponding linear map with respect to the basis $\binom{\mathbb{Z}_n}{k}$. We show that $R_{n,k}$ has rank $\binom{n}{k}$ when k is even and $\binom{n-1}{k-1}$ when k is odd. We prove that the extremal rays of the cone generated by the rows of $R_{n,k}$ are just all its rows. When k is even, we can also determine all facets of this cone. We list part of these results in a more precise way below.

Assume that $1 < k < n$. For $A \in \binom{\mathbb{Z}_n}{k}$ and $B \in \binom{[n-1]_n}{k}$, put

$$f_n(A) := \sum_{j \in \mathbb{Z}_n \setminus A} (-1)^{|[j]_n \cap A|} (\{j\} \cup A) \in \mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$$

and

$$g_n(B) := B + \sum_{j \in B} (-1)^{|[j]_n \cap B|} ((B \setminus \{j\}) \cup \{n\}) \in \mathbb{R}^{\binom{\mathbb{Z}_n}{k}}.$$

For $C = \{c_1, \dots, c_k\} \in \binom{\mathbb{Z}_n}{k}$ and $\epsilon \in \{0,1\}^C$, let $C_\epsilon := \{c_1 - \epsilon_{c_1}, \dots, c_k - \epsilon_{c_k}\} \in \left(\!\binom{\mathbb{Z}_n}{k}\!\right)$, and

$$h_n(C) := \sum_{\epsilon \in \{0,1\}^C, C_\epsilon \in \binom{\mathbb{Z}_n}{k}} (-1)^{\sum_{i \in C} \epsilon_i} C_\epsilon \in \mathbb{R}^{\binom{\mathbb{Z}_n}{k}}.$$

We view $\mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$ as an Euclidean space with $\binom{\mathbb{Z}_n}{k}$ as a standard normal basis and we write $\langle \cdot, \cdot \rangle$ for the corresponding inner product. The next result gives an expression of the inverse matrix of $R_{n,k}$ when k is even.

Theorem 1. *When k is even and $1 < k < n$, $\langle R_{n,k}(A), h_n(C) \rangle = 2\delta_{A,C}$ for $A, C \in \binom{\mathbb{Z}_n}{k}$.*

The gap of a state $C \in \binom{\mathbb{Z}_n}{k}$, denoted by $\|C\|$, is given by

$$\|C\| := \min_{(a,b) \in C \times C, a \neq b} |[a,b]_n| - 1.$$

Let $R_{n,k,t}$ denote the submatrix of $R_{n,k}$ obtained by removing all rows indexed by $\{C : \|C\| < t\}$. Note that $R_{n,k,1} = R_{n,k}$. Here is an easy corollary of Theorem 1.

Corollary 1. *When k is even and $1 < k < n$, $\varphi \in \mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$ is a nonnegative linear combination of the rows of $R_{n,k,t}$ if and only if its inner product with $h_n(C)$ is nonnegative when $\|C\| \geq t$ and is 0 when $\|C\| < t$ for all $C \in \binom{\mathbb{Z}_n}{k}$.*

When $(k,t) = (2,1)$, Corollary 1 reduces to the characterization of Kalmanson matrix [2, Theorem 33]; when $(k,t) = (2,2)$, Corollary 1 reduces to the characterization of Kalmanson metrics [2, Theorem 31] [1, Theorem] [3, Theorem 5.2].

Theorem 2. *Assume that k is odd and $1 < k < n$. Then, $\left\{f_n(A) : A \in \binom{[n-1]_n}{k-1}\right\}$ forms a basis of $\operatorname{Im} R_{n,k}$, while both $\left\{g_n(B) : B \in \binom{[n-1]_n}{k}\right\}$ and $\left\{h_n(C) : C \in \binom{[n-1]_n}{k}\right\}$ are bases of $\operatorname{Ker} R_{n,k}$. Moreover, $\operatorname{Im} R_{n,k}$ and $\operatorname{Ker} R_{n,k}$ are orthogonal complements of each other in $\mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$.*

The study of $R_{n,2}$ is related to trees, consecutive-ones property, circular split systems, Kalmanson matrics and Robinsonian matrices. In this ongoing research, we intend to see what happens for general k.

Suppose that $a_1, \ldots, a_k \in \mathbb{Z}_n$ are k different elements such that the interval with head a_i and tail a_{i+1} contains exactly two of these k elements, namely a_i and a_{i+1} for $i = 1, \ldots, k-1$. Define $\mathrm{E}_{n,k}(a_1 \wedge \cdots \wedge a_k) = (\sum_{a \in [a_1, a_2 - 1]} a) \wedge \cdots \wedge (\sum_{a \in [a_{k-1}, a_k - 1]} a) \wedge (\sum_{a \in [a_k, a_1 - 1]} a)$. This induces a well-defined linear map $\mathrm{E}_{n,k}$ from $\wedge^k(\mathbb{R}^{\mathbb{Z}_n})$ to itself. Let Θ_n^k be the linear map from $\mathbb{R}^{\binom{\mathbb{Z}_n}{k}}$ to $\bigwedge^k \mathbb{R}^{\mathbb{Z}_n}$ such that for all $C \in \binom{\mathbb{Z}_n}{k}$, $\Theta_n^k(C) := c_1 \wedge \cdots \wedge c_k$, where $C = \{c_1, \ldots, c_k\}$ and $c_i \in [c_{i+1} - 1]_n$ for all $i = 1, \ldots, k-1$. Given $C \in \binom{\mathbb{Z}_n}{k}$, let $h''_n(C)$ be

$$\sum_{\epsilon \in \{0,1\}^C, C_\epsilon \in \binom{\mathbb{Z}_n}{k}} (-1)^{(k-1)\epsilon_n + \sum_{i \in C} \epsilon_i} \Theta_n^k(C_\epsilon) \in \wedge^k \mathbb{R}^{\mathbb{Z}_n}.$$

provided $n \in C$, and let $h''_n(C)$ be

$$\sum_{\epsilon \in \{0,1\}^C, C_\epsilon \in \binom{\mathbb{Z}_n}{k}} (-1)^{\sum_{i \in C} \epsilon_i} \Theta_n^k(C_\epsilon) \in \wedge^k \mathbb{R}^{\mathbb{Z}_n}$$

otherwise.

Theorem 3. *Both $\{\Theta_n^k \circ f_n(A) : A \in \binom{[n-1]_n}{k-1}\}$ and $\{\mathrm{E}_{n,k} \circ \Theta_n^k(A \cup \{n\}) : A \in \binom{[n-1]_n}{k-1}\}$ are bases of $\operatorname{Im} \mathrm{E}_{n,k}$; while both $\{\Theta_n^k \circ g_n(B) : B \in \binom{[n-1]_n}{k}\}$ and $\{h''_n(C) : C \in \binom{[n-1]_n}{k}\}$ are bases of $\operatorname{Ker} \mathrm{E}_{n,k}$.*

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Integrality of some Cayley graphs

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This is joint work with Daria Lytkina and Viktor Mazurov

A fnite graph is said to be integral if all eigenvalues of its adjacency matrix are integers.

Let S be a nonempty subset of a finite group G such that $1 \notin S$ and if $s \in S$ then $s^{-1} \in S$. *Cayley graph* $\mathrm{Cay}(G, S)$ of G associated with S is a graph whose vertex set is G itself and two vertices $x, y \in G$ are adjacent if and only if there exists $s \in S$ such that $y = xs$.

A subset S of a group G is said to be *normal* if $s \in S$ implies $g^{-1}sg \in S$ for every $g \in G$.

In the talk, we discuss the following problems posed by D.Lytkina in [1, Problem 19.50]:

a) Let G be a finite group generated by a normal subset R consisting of elements of order 2. Is it true that the Cayley graph $\mathrm{Cay}(G, R)$ is integral?

b) Let A_n be the alternating group of degree n, let $S = \{(123), (124), \ldots, (12n)\}$ and $R = S \cup S^{-1}$. Is it true that the Cayley graph $\mathrm{Cay}(A_n, R)$ is integral?

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CI-property for decomposable Schur rings over an elementary abelian group

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Let G be a finite group. A set $S \subseteq G$ is called a CI*-subset* if for every $T \subseteq G$ the isomorphism of Cayley graphs $Cay(G, S)$ and $Cay(G, T)$ implies that $T = S^{\varphi}$ for some $\varphi \in Aut(G)$. A group G is said to be a DCI*-group* if each of its subsets is a CI-subset. In 1978 Babai and Frankl asked which are the DCI-groups. One of the main steps towards the classification of all DCI-groups is the classification of all elementary abelian DCI-groups. The previously obtained results imply that an elementary abelian group of rank at most 5 is a DCI-group . On the other hand, it is known that an elementary abelian group of a sufficiently large rank is not DCI-group.

A Schur ring over a group G is called a CI*-Schur ring* if for every its isomorphism f to a Schur ring over G there exists a Cayley isomorphism which induces the same algebraic isomorphism as f. In [2] Hirasaka and Muzychuk proved the following statement: if every schurian Schur ring over a given group G is a CI-Schur ring then G is a DCI-group. Let p be a prime number and C_p be a cyclic group of order p. The proofs of the fact that the group $G = C_p^n$ is a DCI-group for $n \in \{4, 5\}$ and odd prime p (see [1, 2]) are based on the above result of Hirasaka and Muzychuk. In fact, in these proofs it was checked that every schurian Schur ring over G is a CI-Schur ring. One of the main difficulties here was to check that every decomposable Schur ring over G is a CI-Schur ring. Recall that a Schur ring is called *decomposable* if it is the generalized wreath product of two smaller Schur rings. We establish a sufficient condition for a decomposable Schur ring over an elementary abelian group to be a CI-Schur ring. By using this condition we check in a short way the CI-property for decomposable Schur rings over an elementary abelian group of rank at most 5.

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On uniform partitions of F^n into Hamming codes

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A code C is called *perfect binary single-error-correcting code* (briefly a perfect code) if for any vector x from the set F^n of all binary vectors of length n there exists exactly one vector $y \in C$ at the Hamming distance not more than 1 from the vector x. *The automorphism group of any partition* $P^n = \{C_0, C_1, \ldots, C_n\}$ of the set F^n into perfect codes $C_0, C_1, \ldots, C_n$ of length n, $\bigcup_{i=0}^{n} C_i = F^n$ is defined as the group of isometries of F^n preserving the partition P^n. A partition P^n is called *transitive*, if for any two codes C_i and C_j, i, j from $I = \{0, 1, \ldots, n\}$, there is an automorphism σ from $\mathrm{Aut}(P^n)$ such that $\sigma(C_i) = C_j$. A partition P^n of F^n is defined to be 2-*transitive*, if for any two subsets $\{i_1, i_2\}$ and $\{j_1, j_2\}$ of I there exists an automorphism σ from $Aut(P^n)$ such that $\sigma(C_{i_t}) = C_{j_t}, t = 1, 2$. By definition any 2-transitive partition is transitive. A survey concerning partitions and all other necessary definitions can be found in [1].

A perfect linear code is called the *Hamming code*. Let e_i be a binary vector in F^n of weight 1 with one in the ith coordinate position. A partition $P^n = \{H_0, H_1 + e_1, \ldots, H_n + e_n\}$ of F^n into cosets of Hamming codes $H_0, H_1, \ldots, H_n$ of length n we call *uniform* if any two Hamming codes H_i, H_j, $i, j \in I$, satisfy $\eta_n = |H_i \bigcap H_j| = \text{const}$. Such partitions of F^n into cosets of Hamming codes with the smallest possible size of η_n were constructed for length $n = 7$ by Phelps in [2] and for any $n = 2^m - 1$ for odd $m > 3$, using the Gold function by Krotov in [3].

We give the recursive construction of the class of uniform partitions into Hamming codes exploiting the classical Mollard's construction for perfect codes [4] and the results [2, 3].

Theorem. *For any $n = 2^m - 1$, $m > 2$ and $l = 1, 2, \ldots, [(m+1)/2]$, with the exception $m = 4$, $l = 1$, there exists 2-transitive uniform partition $P^n = \{H_0, H_1 + e_1, \ldots, H_n + e_n\}$ of F^n into cosets of Hamming codes $H_0, H_1, \ldots, H_n$ of length n for η_n satisfying*

$$\log_2(\eta_n) = n - 2m + 2l - \delta(m),$$

where $\delta(m) = \begin{cases} 1 \textit{ for } m \equiv 1(mod\ 2); \\ 0 \textit{ for } m \equiv 0(mod\ 2). \end{cases}$

Remark. It should be noted that this theorem covered a half part of possible values of the numbers η_n. Another part is still open.

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Minimum supports of eigenfunctions in bilinear forms graphs

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Bilinear forms graph $Bil_q(n,m)$ is a distance-regular graph with the vertex set V consisting of all $n\times m$ matrices over a finite field F_q and two vertices being adjacent when their matrix difference has a rank 1. A function $f: V \to \mathbb{R}$ that is not constantly zero and satisfies the equation $\theta f(u) = \sum\limits_{v\sim u} f(v) \quad \forall u \in V$ is called an *eigenfunction* of a graph corresponding to an eigenvalue θ of its adjacency matrix.

We are interested in finding eigensupports of minimum cardinality and describing their structure. This problem was explored for several families of graphs in the works [1–6]. In case of distance-regular graphs there is a well known *weight distribution* lower bound (see [1], for example) for the support cardinality which can be calculated from the intersection numbers of a graph.

This work studies the eigenfunctions of $Bil_q(n,m)$ corresponding to its minimal eigenvalue. The first part is dedicated to the case $n=m=2$ over the prime field F_p. We prove that the weight distribution bound can be achieved and provide an explicit construction that gives rise to a desired eigenfunction with minimum support. This construction is described below:

Theorem 1. *Let a_1 be a generating element of the multiplicative group F_p^*. Denote $a_0 = 0$; $a_2 = a_1^2$; $\dots$; $a_{p-2} = a_1^{p-2}$; $a_{p-1} = a_1^{p-1} = 1$. Choose $\delta \in F_p$, such that $\delta \neq -\xi^2$ for all $\xi \in F_p$. The independent set* $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} \frac{1}{a_i^2\delta+1} & \frac{a_i}{a_i^2\delta+1} \\ \frac{a_i\delta}{a_i^2\delta+1} & \frac{a_i^2\delta}{a_i^2\delta+1} \end{bmatrix}$ *together with the vertices* $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} \frac{1}{a_i^2\delta+1} & \frac{-a_i}{a_i^2\delta+1} \\ \frac{a_i\delta}{a_i^2\delta+1} & \frac{1}{a_i^2\delta+1} \end{bmatrix}$ *, where $i = 0\dots p-1$, form a minimum eigensupport as two parts of a complete bipartite graph $K_{p+1,p+1}$.*

In the second part of the work we are recalling a well known representation of a bilinear forms graph $Bil(n,m)$ as a subgraph of a Grassmann graph $J_q(n+m,m)$. For a Grassmann graph the minimum supports are characterized and can be presented in terms of quadtratic forms and its totally isotropic spaces [7]. Using this important connection we explore the non-existence of eigenfunctions with minimum supports achieving the weight distribution bound in the case of bilinear forms graphs with a diameter at least 3.

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On the numbers of transversals and multiplexes in iterated quasigroups

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A d-ary quasigroup of order n is a d-ary operation over a set of cardinality n such that the Cayley table of the operation is a d-dimensional latin hypercube of order n. Given a binary quasigroup G, the d-iterated quasigroup $G^{[d]}$ is a d-ary quasigroup that is a $(d-1)$-time composition of G with itself.

A k-multiplex K in a d-dimensional latin hypercube Q of order n or in the corresponding d-ary quasigroup is a multiset of kn entries such that each hyperplane and each symbol of Q is covered by exactly k elements of K. 1-multiplexes are mostly known as transversals. Denote by $P_k(Q)$ the number of k-multiplexes in a latin hypercube Q.

We propose a method for counting and constructing all k-multiplexes in iterated quasigroups $G^{[d]}$, assuming that a binary quasigroup G is given. The main consequence of this method is the following theorem.

Theorem 1. *Let G be a binary quasigroup of order n and let $Q(G^{[d]})$ be the d-dimensional latin hypercube corresponding to the d-iterated quasigroup.*

1. *For all odd d the d-dimensional latin hypercube $Q(G^{[d]})$ has a k-multiplex. If for some even d' the hypercube $Q(G^{[d']})$ has a k-multiplex then for all $d \geq d'$ the hypercube $Q(G^{[d]})$ has a k-multiplex.*
2. *There exists a constant $c(G,k) > 0$ for which*

$$\lim_{d\to\infty} \frac{P_k\left(Q(G^{[d]})\right)}{\left(\frac{(kn)!}{k!^n}\right)^{d-1}} = c(G,k),$$

where all d are such that $Q(G^{[d]})$ has a k-multiplex.

The asymptotic behavior of the maximum number of transversals in latin hypercubes of fixed dimension and large order was found in [2,3]. In [1] it was proved that if for an abelian group G the latin hypercube $Q(G^{[d]})$ has a transversal then for large n and fixed d the number of transversals in $Q(G^{[d]})$ asymptotically reaches the upper bound. Theorem 1 implies that the analogous statement holds for the Cayley tables of d-iterated quasigroups of fixed order and large dimension.

In this talk we also characterize a typical k-multiplex in an iterated quasigroup and provide limit constants $c(G,1)$ for the numbers of transversals in several iterated quasigroups of small orders.

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Inverse limits of m-sprouts and topological self-similar dendrites

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Definition. *Let $I = \{1, ..., m\}$ be the index set and $\Gamma = (V, E)$ be a tree such that*
1) *V is divided into 2 parts: $V = B \sqcup W$, $E \subset B \times W$; $\#B \geq m$ and the set of endpoints $B_F \subset B$;*
2) *there is injective map $\nu : I \to B$, and edge coloring $\varphi : E \to I$, injective on each $E(w)$ for any $w \in W$.*

Then the tree $\Gamma = \Gamma(B, W, E, \nu, \varphi)$, is called a m-sprout. Such settings allow to define a composition operation $\Gamma_1 * \Gamma_2$ on the set $Sp(m)$ of all m-sprouts.

There are several objects associated with a m-sprout Γ:
a) connected finite acyclic non-Hausdorff space $X(\Gamma) = (V, \tau)$, where τ is a topology generated by neighbourhoods of "black"points $\{N(b), b \in B\}$ in Γ;
b) a digraph $\mathcal{G}(\Gamma)$ with the vertex set I, called *index diagram of* Γ;
c) two semigroups G_ψ and G_ϕ of maps $\psi_w : I \cup \{0\} \to I \cup \{0\}$(resp. $\phi_w : I \to I$) relating indexed points in $\nu(I)$ to edge indices in $E(w)$.
For $u \in G_\psi$ or $u \in G_\phi$, we define $Inv(u) = \max\{I' \subset I : u(I') = I'\}$.

If $\Gamma = \Gamma_1 * \Gamma_2$, then for each $w \in W_1$ we have an isomorphic embedding $f_w : \Gamma_2 \to \Gamma$ such that $\Gamma = \bigcup\limits_{w \in W_1} f_w(\Gamma_2)$, which restricts to $f_w : X(\Gamma_2) \to X(\Gamma)$, giving the representation $X(\Gamma) = \bigcup\limits_{w \in W_1} f_w(X(\Gamma_2))$. There are also a natural embedding $J : B(\Gamma_1) \to B(\Gamma)$ and projection $\pi : X(\Gamma) \to X(\Gamma_1)$ such that $\pi \circ J = Id|_{B_1}$.

Let Γ_1 be a m-sprout, and put $\Gamma_n = \Gamma_1^n$, and denote by $X_n = X(\Gamma_n)$ the associated topological space. Consider the sequence of projections: $X_1 \xleftarrow{\pi_{1,1}} X_2 \xleftarrow{\pi_{2,1}} ... \xleftarrow{\pi_{n-1,1}} X_n \xleftarrow{\pi_{n,1}} ...$ and let its inverse limit be $X = \varprojlim X_n$. Under certain conditions on the semigroup G_ψ, X is Hausdorff. The space X satisfies the equation $X = \bigcup\limits_{w \in W_1} f_w(X)$, so X is self-similar with respect to the system $\mathcal{S} = \{f_{w_i}, w_i \in W_1\}$. Since all X_n are acyclic connected quasi-compact spaces, the same is true for X. We prove that X is a dendrite, find the conditions of finiteness of its ramification order and show that the arcs, connecting the points in $\nu(I) \subset B$, are the components of an attractor of a graph-directed system:

Theorem 1. *If for any $u \in G_\psi$, then $\#Inv(u) \leq 1$ X is a dendrite.*

Theorem 2. *If the index diagram of Γ does not contain cyclic vertices with outgoing ramification order ≥ 2, then the ramification order for the points of X is bounded.*

Theorem 3. *For any $b, b' \in \nu(I)$ there are unique s and s-tuples $j_1, ..., j_s$, $k_1, ..., k_s$ and $l_1, ..., l_s$ so that*

$$\gamma_{bb'} = \bigcup_{i=1}^{s} f_{w_{j_i}}(\gamma_{b_{k_i} b_{l_i}})$$

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On vertex-transitive antipodal distance-regular graphs of diameter three with primitive almost simple antipodal groups

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We consider the problem of classification of antipodal distance-regular graphs of diameter three satisfying the following condition: there is a group of automorphisms G of the graph, which acts transitively on its vertices and induces an almost simple primitive permutation group G^{Σ} (which will be also referred to as an antipodal group of the graph) on the set Σ of its antipodal classes.

This problem has been recently solved in the class of arc-transitive graphs (see [1–3]), in particular, arc-transitive antipodal distance-regular graphs of diameter three possess 2-transitive antipodal groups and form a quite rich class of graphs which consists of families of distance-transitive graphs together with four infinite families of arc-transitive but not distance-transitive graphs. However, very little is known in general case. Non-arc-transitive examples are provided by two Klin-Pech graphs with intersection arrays $\{35, 24, 1; 1, 12, 35\}$ and $\{44, 24, 1; 1, 12, 44\}$ with G involving the exceptional 3-cover of A_6 acting vertex-transitively and $G^{\Sigma} \simeq \mathrm{Aut}(A_6)$ being of low permutation rank (≤ 5) in both cases.

Our aim is to solve this problem under the following additional conditions: permutation rank of G^{Σ} is not greater than 5 and $|\Sigma| \leq 2500$. In the talk, we will explore the structure of G and present feasible parameters for each antipodal distance-regular graph of diameter three with the above mentioned properties.

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On eigenfunctions of Hamming graphs with minimum support

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This is joint work with Konstantin Vorob'ev

Denote by $H(n,q)$ the Hamming graph with parameters n and q. It is well-known that the set of eigenvalues of the adjacency matrix of $H(n,q)$ is $\{\lambda_i = n(q-1) - qi \mid i = 0, 1, \ldots, n\}$. Denote by $U_j(n,q)$ an eigenspace corresponding to λ_j. The space $U_i(n,q) + \ldots + U_j(n,q)$ for $i \leq j$ is denoted by $U_{[i,j]}(n,q)$.

In this work we investigate some extremal properties of eigenspaces of the Hamming graph. We consider the problem of finding the minimum cardinality of the support of eigenfunctions of the Hamming graphs $H(n,q)$. This problem is directly related to the problem of finding the minimum possible difference of two combinatorial objects. In more details, these connections are described in [3], where the minimum cardinality of the support of an eigenfunction of the Grassmann graph with the smallest eigenvalue was found. The problem of finding the minimum size of the support of eigenfunctions was studied for the Johnson graphs in [7], for the Doob graph in [1], for the cubic distance-regular graph in [5] and for the Paley graphs in [2]. For the Hamming graph this problem was investigated in [4, 6]. In this paper we obtain the more general result for the functions from the sum of eigenspaces of the Hamming graph. We prove the following result:

Theorem 1. *Let $f : H(n,q) \longrightarrow \mathbb{R}$, $f \in U_{[i,j]}(n,q)$ and $f \not\equiv 0$. Then the following statements are true:*

1. $|f| \geq 2^i (q-1)^i q^{n-i-j}$ *for* $n \geq i+j$ *and* $q \geq 3$.
2. $|f| \geq 2^i (q-1)^{n-j}$ *for* $i+j > n$ *and* $q \geq 4$.

Moreover, we give a characterization of functions with the minimum cardinality of the support for the first case of theorem and for the second case for $i = j$ and $q > 4$.

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Alphabet lifting construction of equitable partitions of Hamming graphs

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Let $G = (V, E)$ be an undirected graph. A partition $(C_1, \ldots, C_t)$ of the set V is an *equitable partition* if for all $i, j \in \{1, \ldots, t\}$ any vertex of C_i has exactly m_{ij} neighbors in C_j. A matrix $M_{t\times t} = (m_{ij})$ is called the *matrix of parameters* of this partition.

The Hamming graph $H(n, q)$ is a graph whose vertices are all words of length n over the alphabet $\{0, 1, \ldots, q-1\}$. Two vertices are adjacent if and only if they differ in exactly one coordinate position.

Equitable partitions of Hamming graphs were investigated by D.G. Fon-Der-Flaass in case $q = 2$ [1–3]. In this work we propose new construction of equitable partitions of Hamming graphs $H(n, q)$ for $q > 2$.

Theorem 1 (Alphabet lifting construction). *Let $(C_1, \ldots, C_t)$ be an equitable partition of the graph $H(n, q_1)$ with the matrix of parameters M. Define the partition of vertices $(C'_1, \ldots, C'_t)$ of the graph $H(n, q_1q_2)$ as follows:*

$$\forall i \in \{1, \ldots, t\}\ \forall x = (x_1, x_2, \ldots, x_n) \in H(n, q_1q_2)$$

$$(x_1, x_2, \ldots, x_n) \in C'_i \iff (x_1 \bmod q_1, \ldots, x_n \bmod q_1) \in C_i.$$

Then $(C'_1, \ldots, C'_t)$ is an equitable partition with the matrix of parameters $q_2M + n(q_2 - 1)E$, where E is the identity matrix of order t.

Traditionally equitable 2-partitions (partitions with $t = 2$) are of special interest. Some constructions from [1] can be easily generalized for the case $q > 2$. Combining these constructions with the one from the theorem we obtain equitable 2-partitions with new matrices of parameters.

Corollary 1. *Take arbitrary $k, m', m'', b, c \in \mathbb{N}$ and prime p such that*

$$m'' < m',\ b + c = kp^{m'},\ gcd(b, c) = kp^{m''},\ gcd(k, p) = 1.$$

Let k_1 be an arbitrary natural divisor of k and s be an arbitrary natural divisor of one of the numbers $m', m' - 1, \ldots, m' - m''$. Then there exist $n_0 \in \mathbb{N}$ such that $\forall n \geq n_0$ there exist an equitable 2-partition of the graph $H(n, q)$ for $q = k_1p^s$ with the matrix of parameters $\begin{pmatrix} n(q-1) - b & b \\ c & n(q-1) - c \end{pmatrix}$.

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On the volume of a compact hyperbolic antiprism

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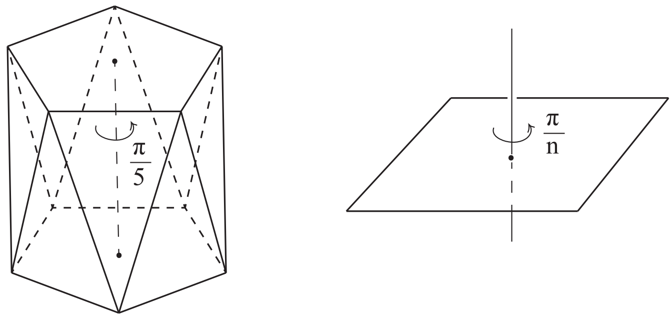
We consider a compact hyperbolic antiprism. It is a convex polyhedron with $2n$ vertices in $\mathbb{H}^3$ which has a symmetry group S_{2n} generated by a mirror-rotational symmetry of order $2n$, i.e. rotation to the angle π/n followed by a reflection. We establish necessary and sufficient conditions for the existence of such polyhedra in hyperbolic space $\mathbb{H}^3$. Then we find relations between their dihedral angles and edge lengths in the form of a cosine rule. Finally, we obtain exact integral formulas expressing the volume of a hyperbolic antiprism in terms of the edge lengths.



n=5

Theorem 1. *A compact hyperbolic antiprism with $2n$ vertices and edge lengths a, c having a symmetry group S_{2n} is exist if and only if*

$$1 + \operatorname{ch} a - 2 \operatorname{ch} c + 2\,(1 - \operatorname{ch} c) \cos\frac{\pi}{n} < 0.$$

Theorem 2. *The volume of a compact hyperbolic antiprism with $2n$ vertices and edge lengths a, c having a symmetry group S_{2n} is given by the formula*

$$V = n \int_{c_0}^{c} \frac{t\,(\operatorname{ch} a - 1)(1 + \operatorname{ch} a + 2\operatorname{ch}^2 t - 4 \operatorname{ch} t \cos\frac{\pi}{n}) + 2\,a\,(\operatorname{ch} t - \cos\frac{\pi}{n}) \operatorname{sh} a \operatorname{sh} t}{(2\operatorname{ch}^2 t - 1 - \operatorname{ch} a)\sqrt{R}}\, dt,$$

where $R = 1 - \operatorname{ch} a\,(2 + \operatorname{ch} a) + 2\operatorname{ch}^2 t + 4\,(\operatorname{ch} a - 1) \operatorname{ch} t \cos\frac{\pi}{n} - 2\,(ch^2 t - 1) \cos\frac{2\pi}{n}$ and c_0 is the root of the equation $1 + \operatorname{ch} a - 2\operatorname{ch} c + 2\,(1 - \operatorname{ch} c) \cos\dfrac{\pi}{n} = 0$.

In particular case $n = 3$ an antiprism become an octahedron with $\overline{3}$-symmetry. In this case theorems 1 and 2 are coincide with the results given in [1]. When $n = 2$ the upper and lower n-gonal faces of an antiprism degenerate to line segments. Thus we get a tetrahedron with a symmetry group S_4. The latter case was studied in [2].

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An expansion property of Boolean multilinear map

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This is joint work with Zongchen Chen

Wu, Xu and Zhu [1] recently introduced the concept of multivariate digraphs. In this talk, we consider an expansion property of Boolean multivariate digraphs. Returning to the usual digraph case, this is to study this problem: If we multiply a Boolean vector by a Boolean matrix repeatedly, how to estimate from below the sizes of the support of the resulting vectors?

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Competition numbers and phylogeny numbers

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This is joint work with Yaokun Wu and Soesoe Zaw

A *graph* G is a pair consisting of its vertex set $\mathrm{V}(G) \neq \emptyset$ and its edge set $\mathrm{E}(G) \subseteq \binom{\mathrm{V}(G)}{2}$. For each graph G and nonnegative integer k, let $\mathrm{I}_k(G)$ stand for the graph obtained from G by adding k isolated vertices. A *vertex-induced subgraph* of a graph G, or simply known as a subgraph of G, is a graph G' such that $\mathrm{V}(G') \subseteq \mathrm{V}(G)$ and $\mathrm{E}(G') = \mathrm{E}(G) \cap \binom{\mathrm{V}(G')}{2}$. Let us write $G' \lhd G$ to mean that G' is a subgraph of G. A digraph D is a pair consisting of its vertex set $\mathrm{V}(D) \neq \emptyset$ and its arc set $\mathrm{A}(D) \subseteq \mathrm{V}(D) \times \mathrm{V}(D)$. For each digraph D, let D° stand for the digraph with $\mathrm{V}(D^\circ) = \mathrm{V}(D)$ and $\mathrm{A}(D^\circ) = \mathrm{A}(D) \cup \{(v, v) : v \in \mathrm{V}(D)\}$. For any $(u, v) \in \mathrm{A}(D)$, we call u an *in-neighbor* of v in D, and call v an *out-neighbor* of u in D. A digraph D is *acyclic* if it contains no cycle.

For every digraph D, the *competition graph* of D [1], denoted by $\mathrm{C}(D)$, is the graph with $\mathrm{V}(\mathrm{C}(D)) = \mathrm{V}(D)$ and with two vertices being adjacent if and only if they have at least one common out-neighbor in D. The *competition number* of a graph G, denoted by $\kappa(G)$, is the least nonnegative integer k such that $\mathrm{I}_k(G)$ becomes the competition graph of an acyclic digraph. Equivalently, $\kappa(G) = \min(|\mathrm{V}(D)| - |\mathrm{V}(G)|)$ where D runs through all acyclic digraphs such that $G \lhd \mathrm{C}(D)$.

For every digraph D, the *phylogeny graph* of D [4], denoted by $\mathcal{P}(D)$, is the competition graph of D°, that is, $\mathcal{P}(D) = \mathrm{C}(D^\circ)$. Note that phylogeny graphs are known as moral graphs in Bayesian network theory [3]. The *phylogeny number* of a graph G, denoted by $\phi(G)$, is the least number p such that we can find a phylogeny graph of an acyclic digraph that has $p + |\mathrm{V}(G)|$ vertices and has G as an induced subgraph.

A *hypergraph* H comprises its vertex set $\mathrm{V}(H) \neq \emptyset$ and its hyperedge set $\mathcal{E}(H) \subseteq \binom{\mathrm{V}(H)}{\geq 2}$. For each hypergraph H and nonnegative integer k, let $\mathrm{I}_k(H)$ stand for the hypergraph with $\mathcal{E}(\mathrm{I}_k(H))$ equals $\mathcal{E}(H)$ and $\mathrm{V}(\mathrm{I}_k(H)) \setminus \mathrm{V}(H)$ is a set of size k. The *subhypergraph* induced by a nonempty subset $A \subseteq \mathrm{V}(H)$ is the hypergraph H' with vertex set A and hyperedge set $\mathcal{E}(H') = \{e \cap A : e \in \mathcal{E}(H), e \cap A \neq \emptyset\}$. For two hypergraphs H and H', we write $H' \lhd H$ to mean that H' is a subhypergraph of H. For every digraph D, the *competition hypergraph* of D [5], denoted by $\mathcal{CH}(D)$, is the hypergraph with vertex set $\mathrm{V}(\mathcal{CH}(D)) = \mathrm{V}(D)$ and hyperedge set

$$\mathcal{E}(\mathcal{CH}(D)) = \{e \in \binom{\mathrm{V}(H)}{\geq 2} : \exists v \in \mathrm{V}(D) \text{ s.t. } e = \{w : (w, v) \in \mathrm{A}(D)\}\}.$$

The ST-*competition number of a hypergraph* H, denoted by $\kappa_{\mathrm{ST}}(H)$, is the least nonnegative integer k such that $\mathrm{I}_k(H)$ becomes the competition hypergraph of an acyclic digraph. Equivalently, $\kappa_{\mathrm{ST}}(H)$ is the least value of $|\mathrm{V}(D) \setminus \mathrm{V}(H)|$ where D runs through all acyclic digraphs satisfying $H \lhd \mathcal{CH}(D)$.

For every digraph D, the ST-*phylogeny hypergraph* of D, denoted by $\mathcal{PH}(D)$, is the competition hypergraph of D°, that is, $\mathcal{PH}(D) = \mathcal{CH}(D^\circ)$. The ST-*phylogeny number of a hypergraph* H, which we write as $\phi_{\mathrm{ST}}(H)$, is the least value of $|\mathrm{V}(D) \setminus \mathrm{V}(H)|$ where D runs through all acyclic digraphs satisfying $H \lhd \mathcal{PH}(D)$.

Theorem 1. *The ranges of the functions $\phi - \kappa + 1$ and $\phi_{ST} - \kappa_{ST} + 1$ are both the set of nonnegative integers.*

Theorem 2. *For any two hypergraphs H_1 and H_2, it holds $\phi_{ST}(H_1 \sqcup H_2) = \phi_{ST}(H_1) + \phi_{ST}(H_2)$, where $H_1 \sqcup H_2$ stands for the disjoint union of H_1 and H_2.*

For any positive integers $m, n_1, \ldots, n_m$, let $[m] = \{1, \ldots, m\}$ and let $K^{n_1,\ldots,n_m}$ denote the graph with

$$\mathrm{V}(K^{n_1,\ldots,n_m}) = \bigcup_{i=1}^{m} \mathrm{V}_i$$

where $\mathrm{V}_i = \{v_i^j : j \in [n_i]\}$ for $i \in [m]$, and with

$$\mathrm{E}(K^{n_1,\ldots,n_m}) = \{v_i^j v_{i'}^{j'} : i \neq i', j \in [n_i], j' \in [n_{i'}]\}.$$

We call $K^{n_1,\dots,n_m}$ a *complete multipartite graph* with m parts and part size $n_1,\dots,n_m$. The *uniform complete multipartite graph*, denoted by K_m^n, is the complete multipartite graph $K^{n,\dots,n}$ with m parts and uniform part size n.

Theorem 3.

(1) $\phi(K_m^2) - \kappa(K_m^2) + 1 = 0$ *for* $m \geq 2$*;*

(2) $\phi(K_m^3) - \kappa(K_m^3) + 1 = 0$ *for* $m \geq 3$*;*

(3) $\phi(K_3^n) - \kappa(K_3^n) + 1 = 0$ *for* $n \geq 2$.

For a graph G, a *clique* of G is a subset of $\mathrm{V}(G)$ such that every two vertices in this subset are adjacent. A clique of G is called *maximal* if it is not properly contained in every clique of G. The *clique hypergraph* of G, denoted by $\mathcal{K}(G)$, is the hypergraph with vertex set $\mathrm{V}(G)$ and with the set of all maximal cliques of G as its hyperedge set. For $G = K^n$, it is easy to see that $\phi_{\mathrm{ST}}(\mathcal{K}(G)) = \kappa_{\mathrm{ST}}(\mathcal{K}(G)) = 0$.

Theorem 4. *Let* $m, n_1, \dots, n_m$ *be positive integers and let* $H = \mathcal{K}(K^{n_1,\dots,n_m})$*. If* $m \geq 2$*, then* $\phi_{ST}(H) + 1 = \kappa_{ST}(H) = \prod_{\ell=1}^{m} n_\ell - \sum_{\ell=1}^{m} n_\ell + m$.

A number of Latin squares of the same order form a set of *mutually orthogonal Latin squares*, often abbreviated in the literature to MOLS, if any two of them are orthogonal. The largest size of a set of MOLS of order n is denoted by $\mathcal{L}(n)$.

Theorem 5. *(Kim-Park-Sano [2, Theorem 3]) Let* m *and* n *be integers such that* $3 \leq n = \mathcal{L}(n) + 1 \leq m$*. Then* $\kappa(K_m^n) \leq n^2 - n + 1$.

Theorem 6. *Let* n *be a positive integer such that* $\mathcal{L}(n) = n - 1$*. Then for every integer* m *bigger than 1, it holds* $\kappa(K_m^n) \leq n^2 - 2n + 2$.

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Tropical hyperplane arrangements and zonotopal tilings

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In tropical geometry, people study the tropical semiring consisting of the real numbers with operations of min and $+$. It has received considerable attention recently due to the strong relationship between several classical questions and their tropical counterparts. Tropical polytopes and tropical hyperplane arrangements have been widely studied [1, 5–7].

Zonotopes are fundamental objects in combinatorial geometry that are closely related to hyperplane arrangements, oriented matroids, and tilings [2,9]. In this paper, we want to study the zonotopal structure arising from tropical hyperplane arrangements. It turns out that zonotopal tropical complexes have nice connections with tree metrics and tropical lines.

For each element $v \in \mathbb{R}^n$, we let $[v]$ denote the equivalence class of v in the tropical projective torus $TT^{n-1} = \mathbb{R}^n/\mathbb{R}\mathbf{1}$ where $\mathbf{1}$ denotes the "all-ones" vector in $\mathbb{R}^n$. These equivalence classes will also be called the "points"of TT^{n-1}. The tropical hyperplane in TT^{n-1} with apex $[v]$ is the set of elements $[x] \in TT^{n-1}$ for which the maximum in $\{x_1 - v_1, \dots, x_n - v_n\}$ is achieved at least twice. Each tropical hyperplane induces a subdivision of TT^{n-1}. Given a discrete set V in TT^{n-1}, we have a tropical hyperplane arrangement whose apices are exactly the points in V. It induces a subdivision of TT^{n-1} that is defined as the common refinement of the subdivisions of TT^{n-1} induced by the tropical hyperplanes in that arrangement, and we denote the subcomplex of bounded faces of this subdivision by $\mathcal{C}_V$. Develin and Sturmfels [1] call $\mathcal{C}_V$ the tropical complex generated by V and observe that the union of faces in $\mathcal{C}_V$ coincides with the tropical convex set generated by V.

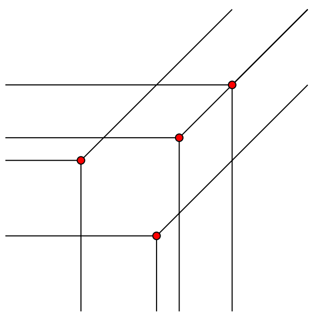

Рис. 1: A tropical hyperplane arrangement in TT^2.

A zonotope is an affine projection of a hypercube. A zonotopal tiling is a subdivision of a polytope by zonotopes and a zonotopal filling is a subdivision of the whole Euclidean space by zonotopes. In Fig. 1, we depict a tropical hyperplane arrangement whose tropical complex is a zonotopal tiling.

Tropical linear spaces are those polyhedral complexes that satisfy the balance condition (also called the zero-tension condition) [4, 7]. We call one-dimensional tropical linear spaces tropical lines. Each tropical line has the shape of an infinite tree.

Theorem 1. *Let V be a finite subset of TT^{n-1}. Then $\mathcal{C}_{-V}$ is a zonotopal tiling if and only if V is on a tropical line T and each ray of T contains at least one point from V.*

Theorem 2. *Let V be an infinite discrete subset of TT^{n-1}. Then $\mathcal{C}_{-V}$ is a zonotopal filling of TT^{n-1} if and only if V is on a tropical line T and each ray of T contains infinitely many points from V.*

Suppose V is a finite subset of a tropical line T such that each ray of T contains at least one point from V. For each ray R of T, we let v_R be the outmost point of R that is in V. We call $Ext(V) := \{v_R \in V : R \text{ is a ray of } T\}$ the extremal point set of V. Let $conv_T(V)$ be the minimum subtree of T containing V. Define T_V to be the subdivision of $conv_T(V)$ by regarding points in V as vertices (dimension zero faces). A partial edge orientation of T_V is called a *cyclic orientation* if all sink and source vertices are in

V and every vertex in $V \setminus Ext(V)$ is on an outgoing arc. Define $CO(T_V)$ to be the poset on all cyclic orientation of T_V ordered under inclusion. See Fig. 2 for an example.

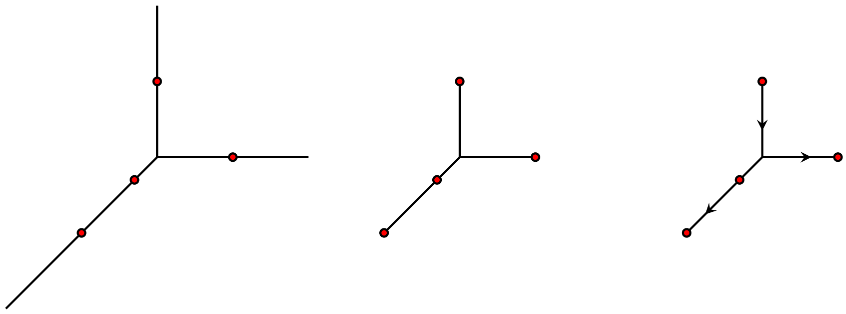

Рис. 2: A cyclic orientation of T_V.

Theorem 3. *Let V be a finite set in TT^{n-1}. If $\mathcal{C}_{-V}$ is a zonotopal tiling, then the face poset of $\mathcal{C}_{-V}$ is anti-isomorphic with $CO(T_V)$, namely there is a bijection α from the face poset of $\mathcal{C}_{-V}$ to $CO(T_V)$ such that A is contained in B if and only if $\alpha(B)$ is contained in $\alpha(A)$ for all faces A and B of $\mathcal{C}_{-V}$.*

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Weakly distance-regular digraphs

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The concept of weakly distance-regular digraphs was firstly introduced by Suzuki and Wang in [2]. Such digraphs have a close relation with association schemes. Some special families of weakly distance-regular digraphs were classified. See [1, 2] for valency 2, [3–5] for valency 3, [1] for thin case and [6] for quasi-thin case.

In this talk, we firstly consider a special family of pseudocyclic association schemes, and obtain the following result.

Theorem 1. Let $(X, \{R\}_{i=0}^{d})$ be an commutative pseudocyclic association scheme generated by a non-symmetric relation R_1 with $p_{1,1}^{1^*} \neq 0$. If

$$\{R_1, R_{1^*}, R_2\} \supseteq R_1^2 \text{ and } \{R_0, R_1, R_{1^*}, R_2, R_{2^*}\} \supseteq R_1 R_{1^*}$$

with $2 \neq 2^*$, then one of the following holds:

(i) $d = 2$ and $|X| = 3$.

(ii) $d = 2$ and $|X| \equiv 3 \pmod 4$.

(iii) $d = 4$ and $|X| = 13$.

As a by-product of Theorem 1, we determine all the primitive weakly distance-regular circulant digraphs in the following result.

Theorem 2. *If Γ is a primitve weakly distance-regular circulant digraph, then Γ is isomorphic to one of the following digraphs:*

(i) *the circuit of length p, where p is prime.*

(ii) *the Paley digraph of p vertices, where $p \equiv 3 \pmod 4$ is prime.*

(iii) $\mathrm{Cay}(\mathbb{Z}_{13}, \{1, 3, 9\})$.

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Group fusion power of association schemes

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There are many ways to construct new association schemes from smaller schemes, say direct product [1], wreath product [2], semidirect product [3,4], crested product [5], wedge product [6], etc. In most cases, the new association scheme is imprimitive. One way to construct new primitive association scheme is the symmetric power (extension) [7]. We generalize the construction of symmetric power, which involves a subgroup G of the symmetric group, and we show that the G-power scheme (defined later) is primitive in most cases.

Let $\mathfrak{X} = (X, \{R_\alpha\}_{\alpha\in\mathcal{I}})$ be an association scheme. Let $G \leq \mathrm{Sym}(d)$ be a subgroup of the symmetric group of degree d. The group G acts on $\mathcal{I}^d$ naturally by permuting the indices, that is $\sigma(\alpha_1, \alpha_2, \ldots, \alpha_d) = (\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \ldots, \alpha_{\sigma(d)})$ for all $\sigma \in G$ and $(\alpha_1, \alpha_2, \ldots, \alpha_d) \in \mathcal{I}^d$. We denote by $\mathcal{O} = \mathcal{I}^d/G$ the orbits under this action. We fuse the two relations in the d-th direct power $\mathfrak{X}^d$ of the association scheme $\mathfrak{X}$, if their corresponding index sequences are in the same orbit of $\mathcal{O}$.

Theorem 1. *The above construction results in a fusion scheme of $\mathfrak{X}^d$.*

We denote the obtained association scheme by $\mathfrak{X}^d/G = (X^d, \{R_\Lambda\}_{\Lambda\in\mathcal{O}})$, and call it the G-power scheme (of $\mathfrak{X}$). Accordingly, we call $\mathfrak{X}$ the base scheme. The direct power and the symmetric power (extension) of association schemes are G-power schemes for $G = \{\mathrm{Id}\}$ and $G = \mathrm{Sym}(d)$ respectively. In fact, it is straightforward from the construction to get the following theorem.

Theorem 2. *Let $H \leq G \leq \mathrm{Sym}(d)$, then $\mathfrak{X}^d/G$ is a fusion scheme of $\mathfrak{X}^d/H$.*

We calculate the intersection numbers of the G-power scheme. And if the base scheme is commutative, we give the Krein parameters, eigenmatrices of the G-power scheme as well. The formulas are omitted here, which are simple combinations of those of power schemes and fusion schemes. Then we give the necessary and sufficient condition for the G-power scheme being primitive.

Theorem 3. *Let $\mathfrak{X}$ be an association scheme and let G be a subgroup of $\mathrm{Sym}(d)$. The G-power scheme $\mathfrak{X}^d/G$ is primitive if and only if G is transitive and $\mathfrak{X}$ is primitive and not thin.*

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Relative t-designs on one shell of Johnson association schemes

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The concept of relative t-designs in Q-polynomial association schemes was first introduced by Delsarte [1] in 1977. For any fixed point $u_0 \in \binom{V}{k}$, each nontrivial shell of Johnson association scheme $J(v,k) = (\binom{V}{k}, \{R_r\}_{0 \le r \le k})$ w.r.t. u_0 is known to be a commutative association scheme which is the product of two smaller Johnson association schemes, but it is not a Q-polynomial association scheme anymore. (The r-th shell of $J(v,k)$ is defined by $X_r = \{x \in \binom{V}{k} : |x \cap u_0| = k - r\}$)

In 1999, Martin [3] defined t-designs in the product of Q-polynomial association schemes. In particular, they are called mixed t-designs in [2] if they are defined on the product of two Johnson association schemes. Moreover, he gave a combinatorial interpretation of Delsarte's generalized Assmus-Mattson Theorem for the Johnson association schemes. We generalize the result of Martin [2] by weakening the assumption of t-designs to relative t-designs and obtain the following result.

Theorem 1. *Fix a point $u_0 \in \binom{V}{k}$. Let Y be a subset of $S := X_{r_1} \cup X_{r_2} \cup \cdots \cup X_{r_p}$ and $w : Y \to \mathbb{R}_{>0}$ be a weight function on Y. If (Y,w) is a relative t-design in $J(v,k)$ with respect to u_0 on p shells S, then $(Y \cap X_{r_\nu}, w)$ is a weighted $(t+1-p)$-design in X_{r_ν} for $1 \le \nu \le p$.*

In the theorem above, weighted t-designs in X_r are the weighted generalization of mixed t-designs.

We also study the classification problems of tight 2-, 3- and 4-designs in one shell X_r of Johnson association schemes $J(v,k)$ with small parameters, say $v \le 1,000$. (Tight t-designs are those whose size attains the lower bound)

(1) If $t = 2$, one interesting observation is that, for $\frac{k}{2} \le r \le \frac{v-k}{2}$, we can expect that all such tight objects come from symmetric 2-designs. However, there is one exception with $v = 528$ which we cannot eliminate now.

(2) If $t = 3$, all the known examples with $v \le 1,000$ arise from Hadamard 2-$(4u-1, 2u-1, u-1)$ designs, but it is still open to prove this theoretically.

(3) There exists no tight relative 4-design on one shell for $v \le 1,000$ according to the result of computer search.

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Phase spaces and kernel spaces of transformation semigroups

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Let Ω be a set and $s \in \Omega^\Omega$. We write $\overline{s}$ and s^{-1} for the two maps from 2^Ω to 2^Ω such that $\overline{s}(X) = \{s(x) : x \in X\}$ and $s^{-1}(X) = \{y : s(y) = x\}$ for all $X \in 2^\Omega$. If a is an element and F is a set such that $f(a)$ is well-defined for every $f \in F$, we adopt the notation $F(a)$ for the set $\{f(a) : f \in F\}$.

A transformation semigroup is a pair (S, Ω) where Ω is a set and S is a sub-semigroup of the full transformation semigroup Ω^Ω. The **phase space** of (S, Ω) is the transformation semigroup $(\overline{S}, 2^\Omega)$ where $\overline{S} = \{\overline{s} : s \in S\}$. The **reduced phase space** of (S, Ω) is the transformation semigroup $(\overline{S}, \overline{S}(\Omega))$.

Let $\mathrm{P}(\Omega)$ be the set of all partitions of Ω. For all $s \in \Omega^\Omega$, we write $\tilde{s}$ for the map from $\mathrm{P}(\Omega)$ to $\mathrm{P}(\Omega)$ such that $\tilde{s}(\Pi) = \{s^{-1}(\pi) : \pi \in \Pi\} \setminus \{\emptyset\}$ for each $\Pi \in \mathrm{P}(\Omega)$. The **kernel space** of (S, Ω) is the transformation semigroup $(\tilde{S}, \mathrm{P}(\Omega))$ where $\tilde{S} = \{\tilde{s} : s \in S\}$. Let 0_Ω be the partition of Ω into singleton sets. The **reduced kernel space** of (S, Ω) is the transformation semigroup $(\tilde{S}, \tilde{S}(0_\Omega))$.

We study some problems about phase spaces and kernel spaces. Below are some sample results in this ongoing research.

Theorem 1. *Let (S, Ω) be a transformation semigroup and let p and q be two positive integers such that $p + q \leq |\Omega|$ and $p \geq q$. If $\overline{S}$ acts transitively on $\binom{\Omega}{p}$, then $\overline{S}$ acts transitively on $\binom{\Omega}{q}$.*

Theorem 2. *There is a polynomial-time algorithm to decide whether or not a given transformation semigroup is a reduced phase space.*

We have a counterpart of **Theorem 2** for reduced kernel space under some assumptions. We also propose parameters to measure the deviation of a transformation semigroup from being a reduced phase space or a reduced kernel space.

On the splitness of the prime and solvable graphs for finite simple groups

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A graph is **split** if its vertex set can be partitioned into a clique and an independent set (either set can be empty). Split graphs were introduced independently by Földes and Hammer [1], and by Tyshkevich and Chernyak [2]. Split graphs are a popular subclass of chordal graphs having many interesting properties. In particular, as proved in [3], split graphs can be characterized in terms of their degree sequences.

We consider two similar types of graphs associated with finite groups. Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of $|G|$. The **prime graph** $\mathrm{GK}(G)$ (the **solvable graph** $\mathcal{S}(G)$) of G is a graph with the vertex set $\pi(G)$, in which two different vertices r and s are adjacent if and only if G has a *cyclic* (*solvable*) subgroup of order divisible by rs.

The prime and solvable graphs of finite simple groups are well studied, see, e.g., [4] and [5]. Relying on these results, we investigate splitness of these graphs. If G is an abelian simple group, then $\mathrm{GK}(G)=\mathcal{S}(G)$ is a singleton, and so is split. Thus, we may assume that G is a nonabelian simple group. Easy calculations show that if G is an alternating group, then the graphs $\mathrm{GK}(G)$ and $\mathcal{S}(G)$ are split. Using [6], we check that the prime graph $\mathrm{GK}(G)$ of any sporadic group G is split, and that there are exactly 10 sporadic groups whose solvable graph $\mathcal{S}(G)$ is nonsplit.

Let G be a finite simple group of Lie type. It is easy to see that in most cases the prime graph $\mathrm{GK}(G)$ is not split. However, it turns out that the **compact form** $\mathrm{GK}_\mathrm{c}(G)$ of this graph is split. Here by the compact form Γ_c of a graph Γ we mean the quotient graph $\Gamma/_\equiv$ with respect to the following equivalence relation on the vertex set V_Γ of Γ: we put $u \equiv v$ if and only if u and v are equal or adjacent and have the same neighbourhood for every $u, v \in V_\Gamma$. Note that the compact form of a split graph is split. Our main result is as follows.

Theorem. *The graph* $\mathrm{GK}_\mathrm{c}(G)$ *of any finite simple group* G *is split.*

In general, the compact form $\mathcal{S}_\mathrm{c}(G)$ of the solvable graph for a simple group of Lie type also splits. Nevertheless, there are examples of nonsplitness of such graphs, and due to the positive solution of the Artin conjecture on primitive roots for almost all primes [7], one can construct infinitely many of them.

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The International Conference and PhD-Master Summer School «Graphs and Groups, Representations and Relations» 2018

Monday August 6	Tuesday August 7	Wednesday August 8	Thursday August 9	Friday August 10	Saturday August 11	Sunday August 12
	07:30 – 09:45 Breakfast					
	10:00 – 13:00 Morning Sessions					
	G2R2 - Summer School		G2R2 - Conference	G2R2 - Summer School		G2R2 - Conference
	Chair: Pisanski 10:00 – 10:50 MC 1: Nedela Lecture 1	10:00 – 10:50 MC 1: Nedela Lecture 3	*Chair: T.Ito* 10:00 – 10:50 Invited talk: Ivanov	10:00 – 10:50 MC 1: Nedela Lecture 5	10:00 – 10:50 MC 1: Nedela Lecture 7	*Chair: Mainardis* 10:00 – 10:50 Invited talk: Soicher
9:00 – 18:00 Registration Conference room 4105 Novosibirsk State University	11:00 – 11:50 MC 1: Nedela Lecture 2	11:00 – 11:50 MC 1: Nedela Lecture 4	11:00 – 11:50 Invited talk: Mainardis	11:00 – 11:50 MC 1: Nedela Lecture 6	11:00 – 11:50 MC 1: Nedela Lecture 8	11:00 – 11:50 Invited talk: Goryainov
	11:50 – 12:10 Coffee break					
	Chair: Mednykh 12:10 – 13:00 MC 2: Jones Lecture 1	12:10 – 13:00 MC 2: Jones Lecture 3	*Chair: Ivanov* 12:10 – 13:00 Invited talk: Matsuo	12:10 – 13:00 MC 2: Jones Lecture 5	12:10 – 13:00 MC 2: Jones Lecture 7	12:10 – 13:00 Invited talk: van Dam
	13:00 – 14:50 Lunch					
	14:50 – 19:00 Afternoon Sessions					
	G2R2 - Summer School		G2R2 - Conference	G2R2 - Summer School		G2R2 - Conference
18:00 – 19:00 Excursion NSU dome	14:50 – 15:40 MC 2: Jones Lecture 2	14:50 – 15:40 MC 2: Jones Lecture 4	14:50 – 15:40 Invited talk: Zhang	14:50 – 15:40 MC 2: Jones Lecture 6	14:50 – 15:40 MC 2: Jones Lecture 8	*Chair: Mednykh* 14:50 – 15:40 Invited talk: Pisanski
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	16:00 – 19:00 G2R2 - Conference					
	Chair: Munemasa	*Kovács*	*Soicher*	*Zhang*	*van Dam*	16:00 – 16:50 Invited talk: Buchstaber
	Gavrilyuk	Erokhovets	16:00 – 16:50 Invited talk: Kovács	Zvezdina	Kabanov	
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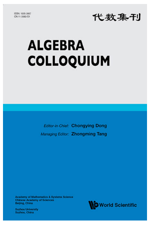
	Yan Zhu	Xu	17:00 – 17:50 Invited talk: Gyürki	Ryabov	Panasenko	17:00 – 17:30 Conference photo
	Mogilnykh	Tetenov		Cho	Vorob'ev	
	18:00 – 18:10 Coffee break					
	Da Zhao	Chanchieva	18:10 – 19:00 Invited talk: Kauffman	Gorshkov	Solov'eva	19:00 – 22:00 Conference dinner
	Brodhead	Drozdov		Fadeev	Yero	
	19:00 – 20:00 Dinner					
	Chair: Abrosimov 20:00 – 22:00 Problem solving MC 1	*Chair: Mednykh* 20:00 – 22:00 Problem solving MC 2	Volleyball	*Chair: Abrosimov* 20:00 – 22:00 Problem solving MC 1	*Chair: Mednykh* 20:00 – 22:00 Problem solving MC 2	

The International Conference and PhD-Master Summer School «Graphs and Groups, Representations and Relations» 2018

Monday August 13	Tuesday August 14	Wednesday August 15	Thursday August 16	Friday August 17	Saturday August 18	Sunday August 19
10:00 – 11:00 Excursion NSU dome 10:00 – 20:00 City Tour 11:30 – 13:00 Football NSU Sports Complex	07:30 – 09:45 Breakfast					
	10:00 – 13:00 Morning Sessions					
	G2R2 - Summer School		G2R2 - Conference	G2R2 - Summer School		G2R2 - Conference
	Chair: Ivanov 10:00 – 10:50 MC 3: Munemasa Lecture 1	10:00 – 10:50 MC 3: Munemasa Lecture 3	*Chair: Wu* 10:00 – 10:50 Invited talk: Koolen	10:00 – 10:50 MC 3: Munemasa Lecture 5	10:00 – 10:50 MC 3: Munemasa Lecture 7	*Chair: Konstantinova* 10:00 – 10:50 Invited talk: T. Ito
	11:00 – 11:50 MC 3: Munemasa Lecture 2	11:00 – 11:50 MC 3: Munemasa Lecture 4	11:00 – 11:50 Invited talk: Gubarev	11:00 – 11:50 MC 3: Munemasa Lecture 6	11:00 – 11:50 MC 3: Munemasa Lecture 8	11:00 – 11:50 Invited talk: Hua
	11:50 – 12:10 Coffee break					
	Chair: Vasil'ev 12:10 – 13:00 MC 4: Muzychuk Lecture 1	12:10 – 13:00 MC 4: Muzychuk Lecture 3	*Chair: Revin* 12:10 – 13:00 Invited talk: Bokut	12:10 – 13:00 MC 4: Muzychuk Lecture 5	12:10 – 13:00 MC 4: Muzychuk Lecture 7	12:10 – 13:00 Invited talk: Gavrilyuk
	13:00 – 14:50 Lunch					13:00 – 13:30 Conference photo
	14:50 – 19:00 Afternoon Sessions					
	G2R2 - Summer School		G2R2 - Conference	G2R2 - Summer School		Closing
	14:50 – 15:40 MC 4: Muzychuk Lecture 2	14:50 – 15:40 MC 4: Muzychuk Lecture 4	14:50 – 15:40 Invited talk: Vdovin	14:50 – 15:40 MC 4: Muzychuk Lecture 6	14:50 – 15:40 MC 4: Muzychuk Lecture 8	
	15:40 – 16:00 Coffee break					
	16:00 – 19:00 G2R2 - Conference					
	Chair: Matsuo	*T.Ito*	*Nedela*	*Wildberger*	*A.Mednykh*	
	Revin	Potapov	16:00 – 16:50 Invited talk: Lando	Kim	Huye Chen	
	Honghai Li	Mogilnykh		Alexander Mednykh	Xiong	
	Valyuzhenich	Sotnikova	17:00 – 17:50 Invited talk: Wildberger	Mattheus	Qian	
	Yang	Taranenko		Vuong	Yinfeng Zhu	
	18:00 – 18:10 Coffee break					
	Dai	Lisitsyna	Open problems session	Mulazzani	Wu	
	Tsiovkina	Parshina		Kamalutdinov	Ilya Mednykh	
	19:00 – 20:00 Dinner					
	Chair: Valyuzhenich 20:00 – 22:00 Problem solving MC 3	*Chair: Ryabov* 20:00 – 22:00 Problem solving MC 4	Volleyball	*Chair: Valyuzhenich* 20:00 – 22:00 Problem solving MC 3	*Chair: Ryabov* 20:00 – 22:00 Problem solving MC 4	



A special issue of Algebra Colloqium “Graphs and Groups, Representations and Relations” (G2R2)

Novosibirsk, Russia, 6–19, 2018

Announcement

Aims&Scope

Algebra Colloquium is an international mathematical journal founded at the beginning of 1994. It is edited by the Academy of Mathematics & Systems Science, Chinese Academy of Sciences, jointly with Suzhou University, and published quarterly in English in every March, June, September and December. Algebra Colloquium carries original research articles of high level in the field of pure and applied algebra. Papers from related areas which have applications to algebra are also considered for publication. This journal aims to reflect the latest developments in algebra and promote international academic exchanges.

Special Issue

Selected papers based on invited and contributed talks given at the International conference on “Graphs and Groups, Representations and Relations” (G2R2), will be published in a special issue of Algebra Colloqium. The topics of special issue cover all fields of pure and applied algebra, and related areas having applications to algebra including algebraic combinatorics and algebraic graph theory.

Guest Editors

Alexander A. Ivanov
Elena V. Konstantinova

Submission

The submission is done directly through `http://123.57.41.99/Jwk_dsjk/EN/volumn/home.shtml`.
When submitting, please indicate that the submission is dedicated to the **Special volume of G2R2**.

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If you have any questions, please contact to the Editorial Office:
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Important dates

Paper submission deadline: November 1, 2018

Groups and Graphs, Designs and Dynamics

Yichang, China, August 12–25, 2019

Announcement

China Three Gorges University organize the International Conference and PhD-Master Summer School on “Groups and Graphs, Designs and Dynamics” (G2D2). All scientific activities will take place in the Mathematical Center at China Three Gorges University, Yichang, China. The scientific program of G2D2 is arranged during August 12 – 25, 2019. Its summer school part consists of 4 short courses and 4 colloquium talks; its conference part consists of about 20 invited talks and some contributed talks.

G2D2 is concerned with all aspects of mathematics, especially those about simple structures and simple processes. We bring together experts and students to exchange ideas and to enrich their mathematical horizon. We organize four short courses and four colloquium talks to let participants see order and simplicity from possibly new perspectives and share insights with experts. We also arrange invited talks (50 minutes) and contributed talks (25 minutes) with topics ranging from coding theory, ergodic theory, graph theory, group theory, optimization theory, quantum information theory, and symbolic dynamics.

The official language of G2D2 is English.

Confirmed Lecturers:

Rosemary A. Bailey and Peter Cameron: Laplacian Eigenvalues and Optimality
University of St Andrews, UK

Mike Boyle and Scott Schmieding: Symbolic Dynamics and the Stable Algebra of Matrices
University of Maryland and Northwestern University, USA

Tullio Ceccherini-Silberstein: Topics in Representation Theory
Universit‘a del Sannio, Italy

Nobuaki Obata: Spectral Analysis of Growing Graphs – A Quantum Probability Point
Tohoku University, Japan