The properties of the set of subarcs of a symmetric irrational dendrite

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Let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be a system of contracting similarities in $\mathbb{R}^2$.
A non-empty compact $K \subset \mathbb{R}^2$ is called *the attractor of the system* $\mathcal{S}$, if $K = \bigcup\limits_{i=1}^{m} S_i(K)$.
The system $\mathcal{S}$ is called *postcritically finite* (PCF), if the set $\{x \in K : \exists i_1 \ldots i_n, j, l : S_{i_1 \ldots i_n}(x) \in K_j \cap K_l\}$ is finite.

It was proved by C. Bandt [1], that K is the attractor of a postcritically finite system $\mathcal{S}$, then the set of dimensions of shortest subarcs $\gamma \subset K$ is finite.

The properties of non-PCF self-similar dendrites are still unexplored.

In the present paper we consider a generalisation of the construction of polygonal dendrites from [2]. We construct a system of four mappings $\mathcal{S} = \{S_1, S_2, S_3, S_4\}$ of an equilateral triangle with vertices $(0;0), (1;0)$ and $(1/2; \sqrt{3}/2)$ to itself, and prove the following proposition.

Proposition 1.
(i) *The system* $\mathcal{S}$ *is not postcritically finite.*
(ii) *The attractor* K *of the system* $\mathcal{S}$ *is a self-similar dendrite.*
(iii) *All subarcs* γ *in* K *have the same Hausdorff dimension* α*.*
(iv) *The set of* α*-dimensional Hausdorff measures of the arcs* γ_{Ox} *with endpoints* $O = (1/2, \sqrt{3}/6)$ *and* $x \in [0,1] \cap K$ *is a self-similar Cantor discontinuum.*

References

[1] C. Bandt, J. Stahnke, Self-similar sets 6. Interior distance on deterministic fractals, preprint. *Greifswald.* (1990).

[2] M. Samuel, A. V. Tetenov, D. A. Vaulin, Self-Similar Dendrites Generated by Polygonal Systems in the Plane. *Sib. Electron. Math. Rep.* **14** (2017) 737–751.