Kernels in 3-regular circulant digraphs

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We are interested in the following open problem in [1]: characterize circulant digraphs which have kernels.

Recall that a kernel J in a digraph D is an independent set of vertices of D such that for every vertex $w \in V(D) \setminus J$, there is an arc from w to a vertex in J.

Subtraction game (also called take-away game) is a well-known impartial combinatorial game (cf. [2], [3], [4], [6]). We use the theory of three elements subtraction game to study the existence of kernels in some 3-regular circulant digraphs, which are directed analogy of circulant graphs (cf. [7]).

The following is the rule of the game. There are a pile of n coins and a set $S = \{a, b, c\}$ of three positive integers, where $a < b < c$. Two players move alternately, subtracting a, b or c from the number of coins. The player who make the last move wins.

Definition (cf. [5]) *A circulant digraph $D = D_n(a, b, c)$ on n vertices with three pairwise distinct jumps a, b, c has vertices $i - a, i - b, i - c \pmod n$ adjacent to each vertex i.*

The main result of the talk is as follows.

Theorem *Suppose that a, b, c are positive integers such that $b \in (a, 2a)$, and $c = m(a + b) + t$, where $m \in \{0, 1, 2, \cdots\}$.*

(1) If m is positive and $t \in [b - a, a)$, then there is a kernel in the circulant digraph $D = D_{b+c}(a, b, c)$.

(2) If $t \in [a, b]$, then there is a kernel in the circulant digraph $D = D_{a+b}(a, b, c)$.

(3) If $t \in (b, a + b]$, then there is a kernel in the circulant digraph $D = D_{a+c}(a, b, c)$.

References

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