Neumaier graphs

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A regular clique, or more specifically an m-regular clique, in a graph Gamma is a clique S such that every vertex of Gamma not in S is adjacent to the same positive number m of vertices of S. In the early 1980s, Neumaier [1] studied regular cliques in edge-regular graphs, and a certain class of designs whose point graphs are strongly regular and contain regular cliques. He then posed the problem of whether there exists a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique.

The first known examples of Neumaier graphs appear in Goryainov and Shalaginov [2], each of which has 24 vertices. In [3], Greaves and Koolen construct an infinite family of Neumaier graphs having 1-regular cliques. Indeed, before our work, all known Neumaier graphs had at least 24 vertices, and contained m-regular cliques only for the value $m = 1$.

In this talk, I will first focus on our determination of the smallest Neumaier graph, which has 16 vertices, valency 9 and a 2-regular 4-clique. Then, I will present a new infinite sequence of Neumaier graphs, the i^{th} element of which contains a 2^i-regular clique.

References

[1] A. Neumaier, Regular cliques in graphs and special 1 1/2-designs, in: Finite Geometries and Designs: Proceedings of the Second Isle of Thorns Conference 1980, Cambridge University Press, Cambridge, 1981, 244–259.

[2] S. V. Goryainov, L. V. Shalaginov, Cayley-Deza graphs with fewer than 60 vertices. *Sibirskie Elektronnye Matematicheskie Izvestiya* **11** (2014) 268–310.

[3] G. R. W. Greaves, J. H. Koolen, Edge-regular graphs with regular cliques. arXiv: 1708.05977v2 (2018). (to appear in European Journal of Combinatorics).