On mixed Moore-Cayley graphs

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The degree-diameter problem is the problem of finding the largest possible number v of vertices of a graph where both the largest degree k of any vertex and the diameter D of the graph are constrained.

In the undirected case, the corresponding upper bound is given by the *Moore* bound:

$$v \leq 1 + k\frac{(k-1)^D - 1}{k-2}, \tag{1}$$

and a graph attaining this bound is called a *Moore* graph [6]. If $D > 1$, then such a graph is regular of degree k with $k \in \{2, 3, 7, 57\}$, while the existence of a graph corresponding to $k = 57$ is a famous open problem, [1, 4]. In the directed case, where we allow only directed arcs in the graph, a bound similar to Eq. (1) can be derived, but there are no non-trivial graphs attaining this bound, [3, 9].

A graph is said to be *mixed* if it contains both undirected edges and directed arcs. Again, a generalization of the Moore bound in Eq. (1) to the case of mixed graphs can be found, however no mixed graph attaining this bound can exist with diameters greater than 2, [8]. Thus, we may focus on mixed graphs of diameter 2, in which case the Moore bound is given by $v \leq (z + r)^2 + z + 1$, where r is the maximum undirected degree of any vertex and z is the maximum directed out-degree. If equality attains, then every vertex of a mixed Moore graph has the same undirected degree r and directed in/out-degree z, and, moreover, $r = (c^2 + 3)/4$ holds for some odd integer c dividing $(4z - 3)(4z + 5)$, [2]. Unlike the case of undirected Moore graphs, there are an infinite number of feasible pairs (r, z), however, only three mixed Moore graphs with $r > 1$ are known: the Bosák graph on 18 vertices, and the two Jørgensen graphs on 108 vertices, [7]. Furthermore, these three graphs are Cayley graphs and therefore it is interesting to search for more examples of mixed Moore graphs that are also Cayley graphs. With the aid of computer, Erskine [5] ruled out further examples of mixed Moore-Cayley graphs for all orders v up to 485.

In this talk, we give a brief survey on the topic and describe an algebraic approach based on the so-called Higman's method in the theory of association schemes, which enables us to rule out the existence of mixed Moore-Cayley graphs of certain orders.

References

[1] E. Bannai, T. Ito, On finite Moore graphs. *Journal of the Faculty of Science, the University of Tokyo. Mathematics* **20** (1973) 191–208.

[2] J. Bosák, Partially directed Moore graphs. *Math. Slovaca* **29(2)** (1979) 181–196.

[3] W. G. Bridges, S. Toueg, On the impossibility of directed Moore graphs. *J. Comb. Theory, Series B* **29(3)** (1980) 339–341.

[4] R. M. Damerell, On Moore graphs. *Mathematical Proceedings of the Cambridge Philosophical Society* **74** (1973) 227–236.

[5] G. Erskine, Mixed Moore Cayley graphs. *Journal of Interconnection Networks* **17(3-4)** (2017).

[6] A. J. Hoffman, R. R. Singleton, On Moore graphs with diameters 2 and 3. *IBM J. Res. Dev.* **4(5)** (1960) 497–504.

[7] L. K. Jørgensen, New mixed Moore graphs and directed strongly regular graphs. *Discrete Math.* **338(6)** (2015) 1011–1016.

[8] M. H. Nguyen, M. Miller, J. Gimbert, On mixed Moore graphs. *Discrete Math.* **307(7)** (2007) 964–970.

[9] J. Plesnik, S. Znám, Strongly geodetic directed graphs. *Acta FRN Univ. Comen.-Math.* **29** (1974) 29–34.