On triple intersection numbers of association schemes

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Let $\mathfrak{X} = (X, \{R_0, \ldots, R_D\})$ be a symmetric association scheme of D classes. For a 3-tuple xyz of points of X, let $[\ell, m, n] := [\ell, m, n]_{x,y,z}$ denote the *triple intersection number* (with respect to xyz) defined by:

$$[\ell, m, n] = \#\{w \in X \mid wR_\ell x, wR_m y, wR_n z\}.$$

Unlike the intersection numbers, the triple intersection numbers $[\ell, m, n]$ depend, in general, on the choice of x, y, z. On the other hand, vanishing of some of the Krein parameters of $\mathfrak{X}$ often leads to non-trivial linear Diophantine equations involving triple intersection numbers as the unknowns (perhaps, it was first observed by Cameron, Goethals and Seidel in [2], see also [1, Theorem 2.3.2]). This fact has been used to compute triple intersection numbers of certain putative distance-regular graphs with feasible intersection arrays, from which non-existence of the corresponding graphs has been shown, see [3–5]. An implementation of this approach is now available as a part of a package for the Sage computer algebra system for checking feasibility of a given intersection array of a distance-regular graph, [6].

Recently, Williford [7] has published lists of feasible Krein parameters for primitive 3-class Q-polynomial association schemes on up to 2800 vertices, and for Q-bipartite (but not Q-antipodal) 4- and 5-class association schemes on up to 10000 and 50000 vertices, respectively. In this work, by computing triple intersection numbers, we rule out many open cases from these lists. If time permits, we will discuss a generalization of triple intersection numbers to quadruples.

References

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