Several results on cliques in strongly regular graphs

Sergey Goryainov
Shanghai Jiao Tong University, China,
Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg, Russia
44g(at)mail.ru

This is joint work with Rosemary Bailey, Peter Cameron, Rhys Evans, Alexander Gavrilyuk, Vladislav Kabanov, Dmitry Panasenko, Leonid Shalaginov, Alexander Valuzhenich

An equitable t-partition of a graph Γ is a partition of the vertex set of Γ into t parts $P_1, \ldots, P_t$ such that, for all $i, j \in \{1, ..., t\}$, every vertex of P_i is adjacent to the same number, namely, p_{ij}, of vertices of P_j. The matrix $\Pi := (p_{ij})_{i,j=1,\ldots,t}$ is called the quotient matrix of the equitable t-partition.

In [2], a new family of maximal cliques was found in Paley graphs of square order, and it was proved that the new family of cliques comes from an equitable partition. In this talk we discuss this result and its possible generalizations.

It is well known that every eigenvalue of Π is an eigenvalue of the adjacency matrix of Γ. In [1], equitable partitions of Latin-square graphs, whose quotient matrix does not contain the eigenvalue -3, were classified. In this talk we discuss a role of maximal cliques in this classification.

An m-regular clique, in a graph Γ is a clique S such that every vertex of Γ not in S is adjacent to the same positive number m of vertices of S (equivalently, S forms a part of an equitable 2-partition). In the early 1980s, Neumaier [3] studied regular cliques in edge-regular graphs, and a certain class of designs whose point graphs are strongly regular and contain regular cliques. He then posed the problem of whether there exists a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique. We thus define a Neumaier graph to be a non-complete, edge-regular, non-strongly regular graph containing a regular clique. In this talk we survey recent results on Neumaier graphs, discuss a determination of the smallest Neumaier graph, which has 16 vertices, and present an infinite family of Neumaier graphs, which extends the smallest one.

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References

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