On directed strongly regular graphs

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A *directed strongly regular graph* (DSRG) with parameters (n, k, t, λ, μ) is a regular directed graph on n vertices with valency k such that every vertex is incident with t undirected edges; the number of directed paths of length 2 directed from a vertex x to another vertex y is λ, if there is an arc from x to y and μ otherwise. Clearly, DSRGs represent a possible generalization of (undirected) strongly regular graphs to the directed case. It was introduced by Duval in [1].

In the talk we present a few constructions of DSRGs.

Firstly, we show a construction based on the Cayley table T of a loop (quasigroup with identity). Let Q be a loop of order n. We refer to the symbol appearing in the x-th row and the y-th column in T as xy. Let us denote the lattice square graph $L_2(n)$, i.e. the graph whose vertices correspond to the cells of the Cayley table T of the loop Q and two vertices are adjacent if and only if they are in the same row or column in T. It is known that the lattice square graph $L_2(n)$ is an undirected strongly regular graph with parameters $(n^2, 2n-2, n-2, 2)$.

By taking two or three copies of $L_2(n)$, respectively, and defining adjacencies between the vertices of different copies, which are based on the operation in the loop Q, we prove the existence of DSRGs with parameter sets $(2n^2, 3n-2, 2n-1, n-1, 3)$, $(2n^2, 4n-2, 2n+2, n+2, 6)$, $(3n^2, 4n-2, 2n, n, 4)$ and $(3n^2, 6n-2, 2n+6, n+6, 10)$ for arbitrary integer n.

Moreover, one of the constructions of DSRGs with parameters $(3n^2, 4n-2, 2n, n, 4)$ seems to be of high interests. It is conjectured that in this case non-isomorphic loops result in non-isomorphic DSRGs. If this conjecture is true, then we have the first evidence of the so-called "prolific construction" of DSRGs, i.e. a construction giving hyper-exponentially many DSRGs with growing n. Similar result is already known in the undirected case of SRGs due to Wallis and Fon-der-Flaass, see [2, 4] and later generalized by Muzychuk [3].

On the other hand it is also conjectured that the (full) group of automorphisms of the DSRG is 6-times larger than the automorphism group of the initial loop. The factor 6 is coming from the permutations of the copies of $L_2(n)$.

Both conjectures were confirmed up to $n \leq 7$ with the aid of a computer.

In the rest of the talk we show constructions of DSRGs which use the technology of the lifts, or voltage assignments.

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References

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