

Prolific construction of strictly Deza graphs

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A non-empty k -regular graph Γ on n vertices is called a *Deza graph* if there exist constants b and a such that any pair of distinct vertices of Γ has either b or a common neighbours. We assume further that $b \geq a$. The quantities n , k , b , and a are called the parameters of Γ and are written as the quadruple (n, k, b, a) .

The concept of Deza graphs was introduced in 1999 by M. Erickson, S. Fernando, W. Haemers, D. Hardy, and J. Hemmeter in the seminal paper [1] influenced by A. Deza and M. Deza [2]. Deza graphs generalize strongly regular graphs in the sense that the number of common neighbours of any pair of vertices in a Deza graph does not depend on their adjacency.

A strongly regular graph has diameter 2, except for the trivial case of a disjoint union of complete graphs. Unlike the strongly regular graphs, Deza graphs can have diameter greater than 2. If a Deza graph has diameter 2 and is not strongly regular, then it is called a *strictly Deza graph*.

In [1] a basic theory of strictly Deza graphs was developed and several ways to construct such graphs were introduced. Some other links to bibliography on Deza graphs could be found in the homepage of M. Deza [3].

W.D. Wallis proposed in [5] a new construction of strongly regular graphs based on an affine design and a Steiner 2-design. Later D.G. Fon-Der-Flaass found how to introduce a sort of randomness into Wallis construction. In [4] he built hyperexponentially many strongly regular graphs with the same parameters.

We show how to modify W.D. Wallis and Fon-Der-Flaass ideas in order to get a new prolific constructions of strictly Deza graphs.

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References

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