Counting Colorings of Cubic Graphs via a Generalized Penrose Bracket

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A *proper edge coloring* of a cubic graph is a coloring of the edges of the graph using three colors so that three distinct colors appear at each node of the graph. It is well-known that the four-color theorem is equivalent to the statement that every isthmus-free planar cubic graph has at least one proper edge coloring. In [1] Roger Penrose gave a graphical recursion formula, *the Penrose Bracket,* that can be seen to count the number of proper edge colorings of a planar cubic graph. The Penrose Bracket does not count the number of colorings of non-planar cubic graphs. For example, the original Penrose Bracket vanishes on the graph $K_{3,3}$ while this graph has 12 proper edge colorings. In this talk we extend the Penrose Bracket to include any non-planar cubic graph [2] so that the new formula counts the number of proper edge colorings of that graph. The method we use can be explained in the original Penrose context of abstract tensors. We use an immersion into the plane of the (possibly) non-planar graph, and we associate a new tensor to each immersion crossing as well as associating an epsilon tensor to each cubic node of the graph. The result is a new state summation formula that correctly counts the number of colorings of the graph. We will discuss the possible applications of this new Penrose Bracket to map coloring and we will discuss related ways to examine the colorings of cubic graphs. We shall discuss the relationships of this work with knot theory and virtual knot theory [3–5].

Acknowledgments. The work has been supported by the Laboratory of Topology and Dynamics, Novosibirsk State University (contract no. 14.Y26.31.0025 with the Ministry of Education and Science of the Russian Federation).

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