Recent progress on graphs with fixed smallest eigenvalue

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In 1976, Cameron, Goethals, Seidel and Shult showed that any connected graph with smallest eigenvalue at least -2 is a generalized line graph or has at most 36 vertices. One year later, in 1977, Hoffman showed that for $\tau > -1-\sqrt{2}$ any connected graph with smallest eigenvalue at least τ and large enough minimal degree is a generalized line graph and hence its smallest eigenvalue is at least -2. The proof of the result of Cameron et al. uses the classification of the root lattices, whereas Hoffman did not need that. But he had to assume a large minimal degree. In this talk I will explain how to generalize the result of Hoffman to graphs with smallest eigenvalue at least -3 using the natural lattice associated to a graph. On the other hand, I will also show that the result of Cameron et al. is much harder to generalize to graphs with smallest eigenvalue at least -3. This is based on joint work with Akihiro Munemasa (Tohoku University), Masood Ur Rehman (USTC), Qianqian Yang (USTC) and Jaeyoung Yang (Anhui University).