Skew-morphisms and regular Cayley maps for dihedral groups

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This is joint work with Young Soo Kwon

Let G be a finite group having a factorization $G = AB$ into subgroups A and B with B cyclic and $A \cap B = 1$, and let b be a generator of B. The associated skew-morphism is the bijective mapping $f : A \to A$ well defined by the equality $baB = f(a)B$ where $a \in A$. The motivation of studying skew-morphisms comes from topological graph theory. A Cayley map M for a group G is an embedding of a connected Cayley graph Γ over G such that the left translations of G become also map automorphisms. We say that M is regular if its automorphism group $\mathrm{Aut}(M)$ acts regularly on the arcs of Γ. In this case $\mathrm{Aut}(M)$ factorizes as $\mathrm{Aut}(M) = G_\ell Y$, where G_ℓ is the group of all left translations of G and Y is cyclic; in particular, every generator of Y induces a skew-morphism of G. Conversely, knowing all skew-morphisms of G is sufficient to know all regular Cayley maps for G. The term 'Cayley map' appeared first in a paper of Biggs in 1972, and since then Cayley maps have become a well established research topic in algebraic and topological graph theory.

The complete classification of regular Cayey maps for cyclic groups were given by Conder and Tucker [1], and this was the first classification result involving an infinite family of groups. Their approach is group theoretical. Using a result of Conder and Isaacs about the commutator subgroup of a product of an abelian group with a cyclic group, they describe first the possible structure of the automorphism group of the maps, and then sort out those groups which give rise to a map.

In this talk, I discuss regular Cayley maps for dihedral groups. Our approach is partly group theoretical and partly relies on skew-morphism techniques. Let D_n denote the dihedral group of order $2n$ and let C_n be a cyclic subgroup of order n. Using a result of Kovács, Marušič and Muzychuk about groups of order at most $2n^2$ containing D_n and in which C_n is core-free, we classify first the core-free maps. We say that a regular Cayley map M for D_n is core-free if $(C_n)_\ell$ is core-free in $\mathrm{Aut}(M)$. Then, using the fact that an arbitrary map is a cover of a core-free map, we derive several constraints on the skew-morphism induced by the map, which will allow us to find eventually the skew-morphism (and therefore the map as well). The results presented in this talk were obtained together with Young Soo Kwon and can be found in [2–4].

References

[1] M. Conder, T. Tucker, Regular Cayley maps for cyclic groups. *Trans. Amer. Math. Soc.* **366** (2014) 3585–3609.

[2] I. Kovács, Y. S. Kwon, Regular Cayley maps on dihedral groups with the smallest kernel. *J. Algebraic Combin.* **44**(4) (2016) 831–847.

[3] I. Kovács, Y. S. Kwon, Classification of reflexible regular Cayley maps for dihedral groups. *J. Combin. Theory Ser. B* **127** (2017) 187–204.

[4] I. Kovács, Y. S. Kwon, Classification of regular Cayley maps for dihedral groups. *preprint.*