Delta-matroids and Vassiliev invariants

Sergey Lando
National Research University Higher School of Economics,
Skolkovo Institute of Science and Technology, Moscow, Russia
lando(at)hse.ru

Vassiliev (finite type) invariants of knots can be described in terms of weight systems. These are functions on chord diagrams satisfying so-called 4-term relations. There is also a natural way to define 4-term relations for abstract graphs, and graph invariants satisfying these relations produce weight systems: to each chord diagram its intersection graph is associated. The notion of weight system can be extended from chord diagrams, which are orientable embedded graphs with a single vertex, to embedded graphs with arbitrary number of vertices that can well be nonorientable. These embedded graphs are a tool to describe finite order invariants of links: the vertices of a graph are in one-to-one correspondence with the link components. We are going to describe two approaches to constructing analogues of intersection graphs for embedded graphs with arbitrary number of vertices. One approach, due to V. Kleptsyn and E. Smirnov, assigns to an embedded graph a Lagrangian subspace in the relative first homology of a 2-dimensional surface associated to this graph. Another approach, due to S. Lando and V. Zhukov, replaces the embedded graph with the corresponding delta-matroid, as suggested by A. Bouchet in 1980's. In both cases, 4-term relations are written out, and Hopf algebras are constructed. Vyacheslav Zhukov proved recently that the two approaches coincide.