On perfect 2-colorings of infinite multipath graphs

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This is joint work with Olga Parshina

A coloring of vertices of a graph G with two colors (black and white) is called *perfect coloring* with parameter matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, if every white vertex has exactly a white neighbors and b black ones, parameters c and d are defined analogously for black vertices.

Consider *an infinite path graph* C_∞, whose set of vertices is the set of integers, and two vertices are adjacent, if they are on the distance 1.

Let G be a transitive graph. We put a copy of graph G into each vertex of the graph C_∞, add edges between every two vertices from neighboring copies. We call this graph *an infinite G-times path* and denote $C_\infty \cdot G$. The graph defined above is exactly the lexicographical product of the graph C_∞ and the graph G. Let n be a positive integer. The subject of the research is perfect 2-colorings of $\overline{K_n}$- and K_n-times paths.

Graphs under consideration have extensive structure, in other words they contain the graph C_∞ as a subgraph. Circulant graphs are similar to them in this sense. Perfect colorings of these graphs are considered in several papers (e.g. see [1]).

Any coloring of the graph $C_\infty \cdot \overline{K_n}$ and $C_\infty \cdot K_n$ is periodic. Denote the period of such a coloring by the $2 \times l$ table, where l is the number of $\overline{K_n}$- or K_n-blocks in this period. Components of first string correspond to numbers of white vertices in such blocks, and components of second string – to numbers of black ones.

Let x, y, z and t be positive integers, that are less than n. Perfect colorings of the $\overline{K_n}$-times path with periods $\begin{pmatrix} x & y & z & t \\ n-x & n-y & n-z & n-t \end{pmatrix}$ under the condition $x + z = y + t$ are called *standard*. *Standard* perfect colorings of the graph $C_\infty \cdot K_n$ are defined as colorings with periods $\begin{pmatrix} x & y & z \\ n-x & n-y & n-z \end{pmatrix}$.

Perfect colorings of these graphs are described in following theorems.

Theorem 1. *Up to renaming of colors, the perfect 2-colorings of the graph $C_\infty \cdot \overline{K_n}$ are exhausted by the standard perfect colorings and two sporadic ones:*

$$\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}, \quad \begin{pmatrix} n & 0 & 0 \\ 0 & n & n \end{pmatrix}.$$

Theorem 2. *Up to renaming of colors, the perfect 2-colorings of the graph $C_\infty \cdot K_n$ are exhausted by the standard perfect colorings and two sporadic ones:*

$$\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}, \quad \begin{pmatrix} n & n & 0 & 0 \\ 0 & 0 & n & n \end{pmatrix}.$$

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References

[1] M. A. Lisitsyna, O. G. Parshina, Perfect colorings of the infinite circulant graph with distances 1 and 2. *J. Appl. Industr. Math* **11**(3) (2017) 381–388.