**Recent developments in Majorana representations of the symmetric groups**

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A major difficulty in studying linear representations of certain finite groups, such as the large sporadics, arises when the degrees of these representations become so large that applying the general methods from linear algebra gets hard, if not practically impossible, even by machine computation.

In this talk I'll cope with a problem which is frequent when dealing with the usual representation of the Monster (and many if its simple subgroups) on the Norton-Conway-Griess algebra, or, more generally, with Majorana representations of finite groups, and can be stated as follows: given a finite group $G$, a complex permutation module $V$ on a finite $G$-set $\mathcal{X}$, and a $G$-invariant positive semidefinite hermitian form $f$, determine the radical $V^\perp$ of $f$ from the Gram matrix $\Gamma$ associated to $f$ with respect to $\mathcal{X}$.

In this context, the $G$-invariance of the form $f$ implies strong restrictions on the Gram matrix $\Gamma$ that can be exploited, via the theory of association schemes, to get a significantly more manageable situation. In fact, $\Gamma$ is equivalent to a block diagonal matrix $\Gamma'$, whose blocks have sizes corresponding to the multiplicities of the irreducible $\mathbb{C}[G]$-submodules of $V$, so that the decomposition of $V^\perp$ into irreducible $\mathbb{C}[G]$-submodules can be recovered from the ranks of the diagonal blocks of $\Gamma'$. The key step to compute the diagonal blocks of $\Gamma'$ is to determine a generaliseation of the first eigenmatrix of the association scheme related to the action of $G$ on $\mathcal{X}$. If this action is multiplicity-free (or, better, if the graph associated to this action is distance transitive), there are well established combinatorial methods to compute this matrix. On the other hand, if the action is not multiplicity-free, this strategy becomes much more awkward, though still possible in some cases: for example, this machinery has been extended to the case where at most one irreducible $\mathbb{C}[G]$-submodule of the complex permutation module on $\mathcal{X}$ has multiplicity 2 and all the others have multiplicity 1.

I'll describe how, in the case of nontransitive actions (which are definitely not multiplicity-free), $V^\perp$ can be determined from the generalised first eigenmatrices of the association schemes related to the actions induced by $G$ on the $G$-orbits of $\mathcal{X}$.