On regular subgroups of the automorphism group of the Hamming code of length 15

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Given a binary vector $x \in F_2^n$ and permutation $\pi \in S_n$ we consider the transformation (x, π), that sends vector $y \in F_2^n$ to $x+\pi(y)$. *The automorphism group* $Aut(F_2^n)$ of F_2^n (bijections preserving Hamming metric) is the set of all such transformations w.t.r composition: $(x, \pi) \cdot (y, \pi') = (x + \pi(y), \pi \circ \pi')$.

The setwise stabilizer of a binary code C of length n in $Aut(F_2^n)$ is called the *automorphism group* $Aut(C)$ of C. If there is a subgroup H of $Aut(C)$ acting transitively and $|H| = |C|$, then H acting on C is called a *regular group* and C is called a *propelinear code* [1].

Obviously, any linear code is propelinear. The Hamming code is known to have the largest order of the automorphism group in the class of perfect binary codes of any fixed length [3] and supposed to have maximum number of regular subgroups of its automorphism group among propelinear perfect codes. However, the fact that the order of the automorphism group of the Hamming code of length n is $|GL(log(n+1), 2)|2^{n-log(n+1)}$ makes attempts of complete classification of regular subgroups impossible for ordinary calculational machinery for smallest nontrivial length 15.

We say that a group $H, H < Aut(F_2^n)$ *is narrow-sense embedded* in a subgroup $G, G < Aut(F_2^n)$, if $H < G$ and $\{\pi : (x, \pi) \in H\} = \{\pi : (x, \pi) \in G\}$.

We considered Nordstrom-Robinson and Hadamard codes that are known to be subcodes of the Hamming code of length 15. These codes have automorphism groups of small order, relatively to that of the Hamming code of length 15 and the classification of regular subgroups of the automorphism groups of both codes was obtained using MAGMA.

Theorem 1. *There are 73 and 39 conjugacy classes of regular subgroups of the automorphism group of Nordstrom-Robinson and Hadamard codes of length 15 respectively, that fall into 45 and 11 isomorphism classes respectively.*

The automorphism groups of Nordstrom-Robinson and Hadamard codes are subgroups that of the Hamming code of length 15, which argues for narrow-sense embedding of regular subgroups.

Theorem 2. *1. The regular subgroups of the Nordstrom-Robinson code are narrow-sense embedded in 605 conjugacy classes of regular subgroups of the automorphism group of the Hamming code of length 15, which fall into at least 219 isomorphism classes. 2. The regular subgroups of the Hadamard code of length 15 are narrow-sense embedded in at least 1207 congugacy classes of regular subgroups of the automorphism group of the Hamming code of length 15, which fall into at least 48 isomorphism classes.*

Remark. We see that there are at least 219 nonisomorphic regular subgroups of the Hamming code of length 15, which significantly exceeds the known lower bounds for the number of nonisomorphic regular subgroups of the automorphism groups of other propelinear perfect codes of length 15, see [2].

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References

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