Integrality of some Cayley graphs

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This is joint work with Daria Lytkina and Viktor Mazurov

A fnite graph is said to be integral if all eigenvalues of its adjacency matrix are integers.

Let S be a nonempty subset of a finite group G such that $1 \notin S$ and if $s \in S$ then $s^{-1} \in S$. *Cayley graph* $\mathrm{Cay}(G, S)$ of G associated with S is a graph whose vertex set is G itself and two vertices $x, y \in G$ are adjacent if and only if there exists $s \in S$ such that $y = xs$.

A subset S of a group G is said to be *normal* if $s \in S$ implies $g^{-1}sg \in S$ for every $g \in G$.

In the talk, we discuss the following problems posed by D.Lytkina in [1, Problem 19.50]:

a) Let G be a fnite group generated by a normal subset R consisting of elements of order 2. Is it true that the Cayley graph $\mathrm{Cay}(G, R)$ is integral?

b) Let A_n be the alternating group of degree n, let $S = \{(123), (124), \dots, (12n)\}$ and $R = S \cup S^{-1}$. Is it true that the Cayley graph $\mathrm{Cay}(A_n, R)$ is integral?

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References

[1] E. I. Khukhro, V. D. Mazurov (editors), *Unsolved Problems in Group Theory, The Kourovka Notebook.* **19** Sobolev Institute of Mathematics, Novosibirsk (2018).