CI-property for decomposable Schur rings over an elementary abelian group

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This is joint work with István Kovács

Let G be a finite group. A set $S \subseteq G$ is called a *CI-subset* if for every $T \subseteq G$ the isomorphism of Cayley graphs $Cay(G,S)$ and $Cay(G,T)$ implies that $T = S^{\varphi}$ for some $\varphi \in Aut(G)$. A group G is said to be a *DCI-group* if each of its subsets is a CI-subset. In 1978 Babai and Frankl asked which are the DCI-groups. One of the main steps towards the classification of all DCI-groups is the classification of all elementary abelian DCI-groups. The previously obtained results imply that an elementary abelian group of rank at most 5 is a DCI-group . On the other hand, it is known that an elementary abelian group of a sufficiently large rank is not DCI-group.

A Schur ring over a group G is called a *CI-Schur ring* if for every its isomorphism f to a Schur ring over G there exists a Cayley isomorphism which induces the same algebraic isomorphism as f. In [2] Hirasaka and Muzychuk proved the following statement: if every schurian Schur ring over a given group G is a CI-Schur ring then G is a DCI-group. Let p be a prime number and C_p be a cyclic group of order p. The proofs of the fact that the group $G = C_p^n$ is a DCI-group for $n \in \{4,5\}$ and odd prime p (see [1,2]) are based on the above result of Hirasaka and Muzychuk. In fact, in these proofs it was checked that every schurian Schur ring over G is a CI-Schur ring. One of the main difficulties here was to check that every decomposable Schur ring over G is a CI-Schur ring. Recall that a Schur ring is called *decomposable* if it is the generalized wreath product of two smaller Schur rings. We establish a sufficient condition for a decomposable Schur ring over an elementary abelian group to be a CI-Schur ring. By using this condition we check in a short way the CI-property for decomposable Schur rings over an elementary abelian group of rank at most 5.

References

[1] Y-Q. Feng, I. Kovács, Elementary abelian groups of rank 5 are DCI-groups. *J. Combin. Theory Ser. A* **157** (2018) 162–204.

[2] M. Hirasaka, M. Muzychuk, An elementary abelian group of rank 4 is a CI-group. *J. Combin. Theory Ser. A* **94** (2001) 339–362.