Cliques and colourings in GRAPE

Leonard H. Soicher
Queen Mary University of London, UK
L.H.Soicher(at)qmul.ac.uk

GAP [2] is an internationally developed, freely available, open-source system for algebra and discrete mathematics. GRAPE [6] is a GAP package for computing with finite graphs with associated groups of automorphisms. In GRAPE, a graph Γ always comes together with an associated subgroup A of the automorphism group of Γ (A can be computed by GRAPE or user-specified), and A is used to store Γ efficiently and to speed up computations with Γ.

For many types of combinatorial objects, the construction or classification problem reduces to finding or classifying cliques with certain properties in a problem-specific graph, where often the graph has a large automorphism group. See, for example, [3, 4, 7]. The GRAPE package provides extensive facilities to exploit graph symmetries for clique finding and clique classification (up to the action of the group A of automorphisms associated with the graph). The general functionality in GRAPE for cliques allows for the classification of the cliques with given vertex-weight sum in a vertex-weighted graph (and the weights can often be non-zero d-vectors of non-negative integers), as well as the finding and classification of cliques invariant under a given group of graph automorphisms. This general clique functionality in GRAPE underpins the functionality in the DESIGN package [5] for the discovery and classification of many types of combinatorial designs.

Recently, I have used the clique functionality in GRAPE to develop programs which exploit the automorphism group of a graph Γ to determine, given a positive integer k, whether Γ has a proper vertex k-colouring, and if so, to produce such a colouring. I have used these programs to determine the primitive permutation groups G of degree at most 255, such that G is a group of automorphisms of a non-null non-complete graph having chromatic number equal to its clique number. These groups are precisely the non-synchronizing primitive permutation groups (of degree at most 255), of interest in both permutation group theory and automata theory (see [1]).

I will talk about these recent developments, and give concrete examples and applications of the clique and colouring machinery in GRAPE, so that hopefully you can apply this machinery to your own research problems.

References

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