## On 2-closures of primitive solvable permutation groups

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Denote by $\Omega$ the set $\{1, \ldots, n\}$, by $Sym_n$ the symmetric group of degree $n$. Denote the action of $Sym_n$ on $\Omega^k$ coordinatewise, i.e. given $\sigma \in Sym_n$ we define

$$\sigma : (x_1, \ldots, x_k) \mapsto (x_1\sigma, \ldots, x_k\sigma).$$

If $G \leq Sym_n$ define the orbits of $G$ on $\Omega^k$ by $\Delta_1(k), \ldots, \Delta_m(k)$. Following H.Wielandt we define the $k$-closure of $G$ (we denote it $G^{(k)}$) by

$$\{\sigma \in Sym_n \mid \Delta_i(k)\sigma = \Delta_i(k) \text{ for } i = 1, \ldots, m\}.$$

In the talk we discuss the possible structure of $G^{(2)}$ for solvable $G \leq Sym_n$.

First we discuss, why we restrict ourself by primitive but not 2-transitive solvable group.

Then we discuss the structure of primitive solvable permutation groups. Namely, if $G \leq Sym_n$ is primitive solvable, then $n = p^k$ for some prime $p$ and $k > 0$. Moreover, $G$ possesses a normal regular elementary abelian subgroup $A$, and if we denote by $L$ a point stabilizer in $G$, then $G = A \rtimes L$.

So we may consider $L$ as a subgroup of $GL_k(p)$ acting on the vector space $F_p^k \simeq A$ in a natural way. It is known (see [1, Theorem 2]) that $A$ is a normal subgroup of $G^{(2)}$, so that if $K$ is a point stabilizer of $G^{(2)}$, then $G^{(2)} = A \rtimes K$. Thus one may assume that $L \leq K$. Furthermore, 2-orbits of $G$ are in one-to-one correspondence with of $L$ in its natural action on $A$. Thus we reduce the original question of finding 2-closure of primitive solvable group to a question of finding 1-closure of irreducible solvable linear groups.

Finally we explain, why we start with primitive linear groups and also give an overview of the results already obtained in this direction.

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### References

[1] C. E. Praeger, J. Saxl, *Closures of finite primitive permutation groups.* Bull. London Math. Soc. **24(3)** (1992) 251–258.