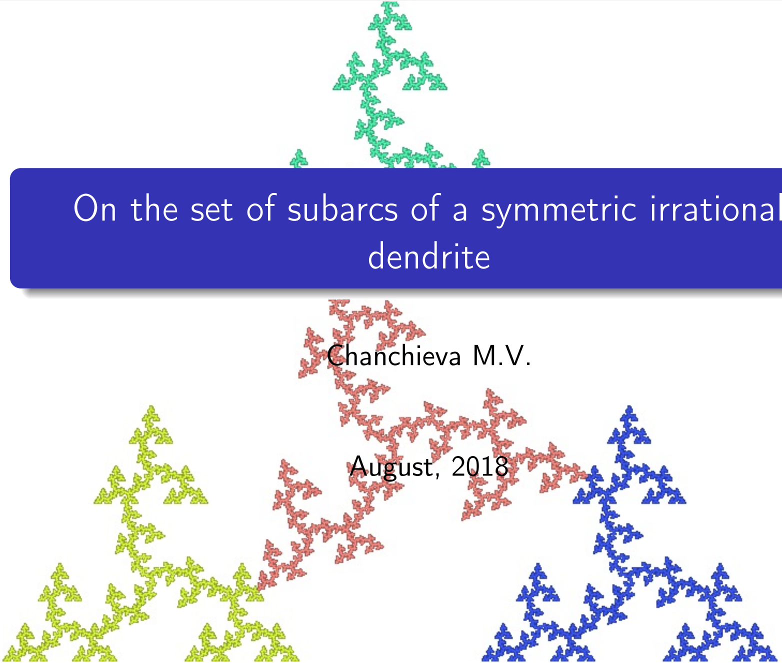

On the set of subarcs of a symmetric irrational dendrite

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Definition

Let $\mathcal{S} = \{S_1, \dots, S_m\}$ be a system of contraction maps of $\mathbb{R}^2$ to itself. A nonempty compact K is called the **attractor** *of the system $\mathcal{S}$, if*

$$K = \bigcup_{i=1}^{m} S_i(K).$$

By Hutchinson's Theorem, the attractor K is uniquely defined by the system $\mathcal{S}$.

Index space I^{∞}

$I = \{1, \dots, m\}$ — the set of indices;
I^n — the set of multiindices $n = i_1 \dots i_n$ of length n;
I^{∞} — the set of infinite sequences $\alpha = i_1 i_2 \dots$ supplied with the infinite product topology.

$$S_{i_1 \dots i_n} = S_{i_1} \circ \dots \circ S_{i_n}$$

$$K_{i_1 \dots i_n} = S_{i_1 \dots i_n}(K)$$

$\alpha \in I^{\infty}$ defines a nested sequence of contractible compact sets

$$K_{i_1} \supset K_{i_1 i_2} \supset K_{i_1 i_2 i_3} \dots$$

Intersection of these sets is a one-point set $\{x\}$.
Defining $\pi(\alpha) = x$ we get the address map $\pi : I^{\infty} \to K$, which is surjective and continuous.

Postcritically finite systems

Let $K = \bigcup\limits_{i=1}^{m} S_i(K)$ be the attractor of a system $\mathcal{S}$ and $\pi : I^\infty \to K$ be the address map.

Critical set of the system $\mathcal{S}$: $\quad \mathcal{C} = \bigcup\limits_{i,j=1}^{m} (K_i \cap K_j)\ , i \neq j$

$\mathcal{S}$ is called **postcritically finite** or PCF, if the set
$\mathcal{P} = \{\alpha \in I^\infty : \exists i_1 \dots i_n : S_{i_1} \dots S_{i_n}(\pi(\alpha)) \in \mathcal{C}\}$ is finite.

Let $z_0 = \pi(i_1 i_2 i_3 \dots)$, put $z_n = \pi(i_{n+1} i_{n+2} i_{n+3} \dots)$. We call z_n the n **predecessor** of z_0.
$\pi(\mathcal{P})$ is the set of all predecessors of the points in $\mathcal{C}$.
If $\mathcal{S}$ is postcritically finite, then $\pi(\mathcal{P})$ is finite.

Contractible P-polygonal systems

Let P be a polygon and V_P be the set of vertices of the polygon P.

Definition

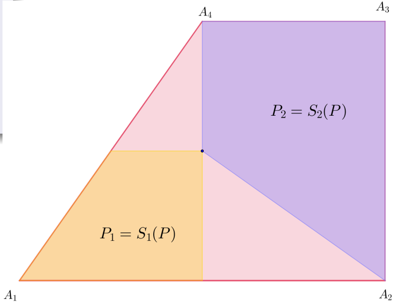
A system of similarities $\mathcal{S}$ is a contractible P-polygonal system, if:

(P1) $P_i = S_i(P) \subset P$, *where* $i \in I$;

(P2) $P_i \cap P_j = V_{P_i} \cap V_{P_j}$, *where* $i, j \in I^\infty$ *end* $i \neq j$;

(P3) $V_P \subset \bigcup\limits_{i \in I} S_i(V_P)$;

(P4) *The set* $\tilde{P} = \bigcup\limits_{i=1}^{m} P_i$ *is contractible.*



Dendrite

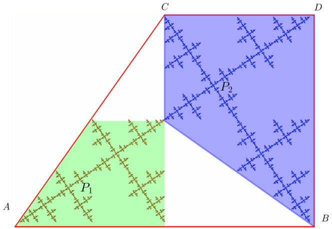
Definition

A **Dendrite** *is a locally connected continuum containing no simple closed curve.*

Theorem

The attractor K of a contractible P-polygonal system of similarities $\mathcal{S}$ is a dendrite.

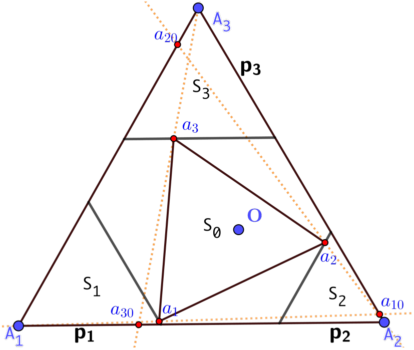
Each P-polygonal system is postcritically finite, because $\pi(\mathcal{P}) = V_P$



Construction

Let $\triangle$ be equilateral triangle with vertices $A_1(0;0)$, $A_2(1;0)$, $A_3(\frac{1}{2};\frac{\sqrt{3}}{2})$, $\mathcal{S}=\{S_0,S_1,S_2,S_3\}$ be a system of contraction similarities, $S_i:\triangle\to\triangle_i$, $S_i(x)=p_i(x-A_i)+A_i$ for $i=\{1,2,3\}$ and $p_1+p_2+p_3\le 1$.

Suppose for some
$\alpha_1=12i_3i_4\ldots,\quad i_n\in\{2,3\};$
$\alpha_2=23j_3j_4\ldots,\quad j_n\in\{1,3\};$
$\alpha_3=31k_3k_4\ldots,\quad k_n\in\{1,2\}.$
the points $a_i=\pi(\alpha_i)$ form an equilateral triangle $\triangle_0:=\triangle_{a_1a_2a_3}$.
Define S_0 by $S_0(A_i)=a_i$, $i=\{1,2,3\}$ and let $\mathrm{fix}(S_0)=O$



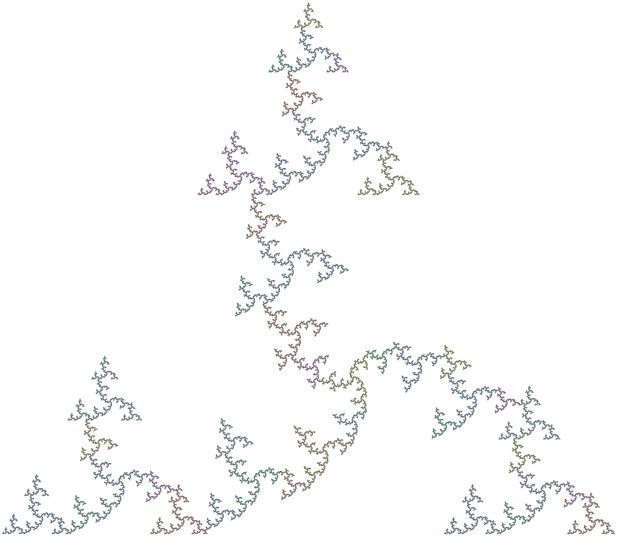
Main Theorem

We prove the following theorem:

Theorem

(1) *The attractor K of the system of similarities $\mathcal{S}$ is a dendrite.*
(2) *If any of the addresses $\alpha_1, \alpha_2, \alpha_3$ is aperiodic then $\mathcal{S}$ is not a polygonal system and is not PCF;*
(3) *Let $d = dim_H(\gamma_{OA_1})$, then for any $x, y \in K$ $\dim_H(\gamma_{xy}) = d$.*
(4) *The set of values of d-dimensional Hausdorff measures $\{H^d(\gamma_{Ox}), x \in K \cap [0,1]\}$ is a self-similar Cantor discontinuum.*

The dendrite



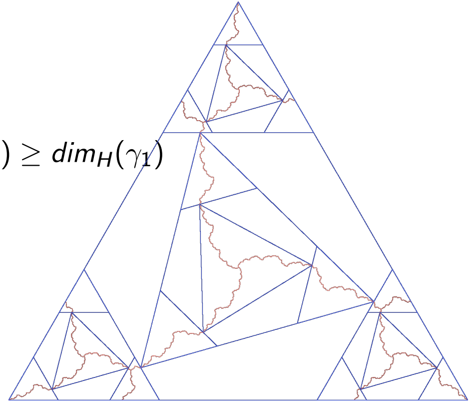
Proposition

All subarcs in K have the same Hausdorff dimension d.

Let $\gamma_i = \gamma_{OA_i}$.
Then γ_i contains $S_i S_0(\gamma_{i+1})$, where by γ_{3+1} we mean γ_1.

$$dim_H(\gamma_1) \geq dim_H(\gamma_2) \geq dim_H(\gamma_3) \geq dim_H(\gamma_1)$$

Then it follows that all they are equal $d = dim_H(\gamma_i)$.
Using this fact and the condition that $p_1 + p_2 + p_3 \leq 1$ we prove that $\forall x, y$ $dim_H(\gamma_{xy}) = d$.



The set of subarcs is self-similar

Proposition

The set of d - dimensional Hausdorff measures of the arcs $\{H^d(\gamma_{Ox}), x \in K \cap [0,1]\}$ *is a self-similar Cantor discontinuum.*

THANK YOU!