Compact n-manifolds via $(n+1)$-colored graphs

Michele Mulazzani

University of Bologna

Graphs and Groups, Representations and Relations
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(joint work with L. Grasselli)

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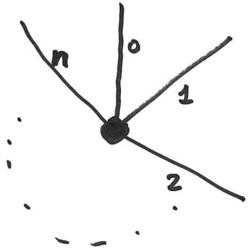
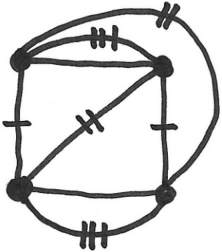
Local model for $(n+1)$-colored graphs:





Left: a 3-colored graph
Right: a 4-colored graph

Residues

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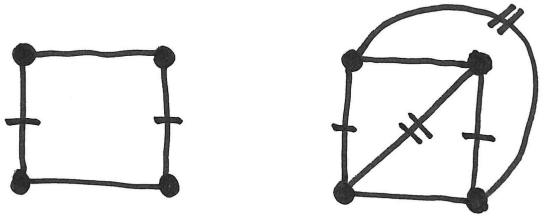

A 2-residue (left) and a 3-residue (right) of the previous graph on the right

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The singular set S_Γ is a subcomplex of $\widehat{M}_\Gamma$ of dimension $\leq n-3$.
If $\dim(S_\Gamma) = 0$ (isolated singularities) then $\widehat{M}_\Gamma$ is a *singular manifold*. .

Characterizing closed manifolds

Proposition (Ferri, 1976)

$\widehat{M}_\Gamma$ is a closed manifold if and only if $\widehat{M}_\Lambda$ is a sphere, for any residue Λ of Γ.

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Remark: all 0-,1- and 2-residues are ordinary.

From n-quasi-manifolds to compact n-manifolds

Suppose $\widehat{M}_\Gamma$ is not a closed manifold (i.e., $S_\Gamma \neq \emptyset$) then we obtain an n-manifold with boundary M_Γ by setting

$$M_\Gamma = \widehat{M}_\Gamma - \mathrm{int}(N(S_\Gamma)),$$

where $N(S_\Gamma)$ is a regular neighborhood of S_Γ in $\widehat{M}_\Gamma$.

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Remark: $\widehat{M}_\Gamma = M_\Gamma$ if and only if $\widehat{M}_\Gamma$ is a closed manifold.

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Remarks on ∂M_Γ

- Connected components of ∂M_Γ correspond to connected components of the singular set S_Γ.

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- If S_Γ is finite (i.e., $\widehat{M_\Gamma}$ is a singular manifold) then ∂M_Γ has no spherical components.

Orientability

Proposition (Cavicchioli – Grasselli – Pezzana, 1980)

$\widehat{M}_\Gamma$ *is orientable* $\Longleftrightarrow \Gamma$ *is bipartite.*

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Corollary (Grasselli – Mulazzani, 2018)

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Existence of representation by colored graphs

Proposition (Gagliardi, 1979)

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Proposition (Casali - Cristofori - Grasselli, 2018)

Any compact manifold without spherical boundary components admits a representation by colored graphs.

Suspension of colored graphs

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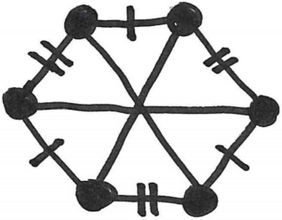
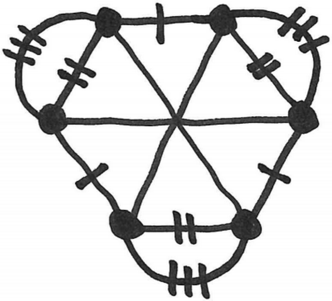
Theorem (Grasselli– Mulazzani, 2018)

- $\widehat{M}_{\Sigma_c(\Gamma)} = \Sigma(\widehat{M}_\Gamma)$;
- $M_{\Sigma_c(\Gamma)} = M_\Gamma \times I$, *if* $\widehat{M}_\Gamma$ *is not a sphere.*

Example: suspension of $S^1 \times S^1$





Left: Γ ($\widehat{M}_\Gamma = M_\Gamma = S^1 \times S^1$)

Right: $\Sigma_2(\Gamma)$ ($\widehat{M}_{\Sigma_2(\Gamma)} = \Sigma(S^1 \times S^1)$, $M_{\Sigma_2(\Gamma)} = S^1 \times S^1 \times I$)

Applications of suspension theorem

Corollary (Grasselli– Mulazzani, 2018)

Let Γ be an $(n+1)$-colored graph.

- *If Γ has order two then $\widehat{M}_\Gamma = M_\Gamma = S^n$.*
- *If Γ has order four and is bipartite then $\widehat{M}_\Gamma = M_\Gamma = S^n$.*
- *If Γ has order four and is not bipartite then $\widehat{M}_\Gamma = \Sigma^{n-2}(RP^2)$ and $M_\Gamma = RP^2 \times B^{n-2}$.*

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Classification results: $n = 3$, closed case

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Orientable closed 3-manifolds:

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Non-orientable closed 3-manifolds:

$c_g \leq 30$, [M.R. Casali – P. Cristofori, 2008], there are 16 non-orientable irreducible RP^2-irreducible closed manifolds.

Classification results: $n = 3$, case with boundary

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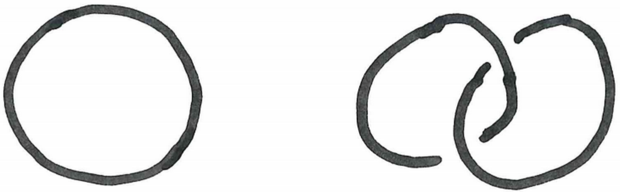
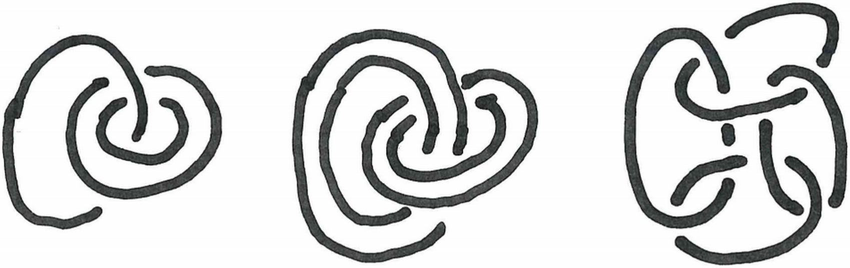
Orientable 3-manifolds with boundary:

$c_g \leq 8$, [P. Cristofori – M. Mulazzani, 2016], there are 5 orientable manifolds with non-empty boundary (and all of them are complements of links in S^3).



The trivial knot and the Hopf link $H = L2a1$



$H\#H$, $L8n8$ and $L8n7$

Classification results: $n = 3$, case with toric boundary

Orientable 3-manifolds with toric boundary:

$c_g \leq 14$, [P. Cristofori – E. Fominykh – M. Mulazzani – V. Tarkaev, 2018], there are 58 orientable irreducible boundary-irreducible manifolds with toric boundary (and 52 of them are complements of links in S^3).

Orientable prime 3-manifolds with toric boundary with $c_g \leq 12$

Name	Code	Manifold	Link
6_1^1	CABCBABCA	$D^2 \times S^1$	unknot
6_1^2	CABCABBCA	$D_1^2 \times S^1$	L2a1
8_1^3	DABCDCABCADB	$D_2^2 \times S^1$	L6n1
8_1^4	DABCDCABCBDA	$(D_2^2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1)$	L8n8
8_2^4	DABCCDABBCDA	`t12047`	L8n7
10_1^2	EABCDDCEABCDEBA	$(D_1^2, (2,1))$	L4a1
10_1^3	EABCDECDABCDEBA	$(D_1^2, (2,1)) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1)$	L10n93
12_1^1	CABFDEFEDCBAEFABCD	$(D^2, (2,1), (2,1))$	–
12_1^2	DABCFEFAECDBBEDFAC	$(M_1^2, (1,0))$	–
12_1^3	FABCDEEDFBACDFEACB	`s776`	L6a5
12_1^4	EABCDFFBEADCEFCABD	$D_3^2 \times S^1$	see fig. ??
12_2^4	EABCDFFDEACBBEADFC	$(D_1^2, (2,1)) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_3^2 \times S^1)$	see fig. ??
12_3^4	EABCDFFDAEBCDCEFBA	$(D_2^2 \times S^1) \bigcup_{\begin{pmatrix}1&2\\1&1\end{pmatrix}} (D_2^2 \times S^1)$	L14n62853
12_4^4	EABCDFFEDABCCDEFAB	$(D_1^2, (2,1)) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1)$	see fig. ??
12_5^4	FABCDEFDAEBCDBEFCA	$(D_2^2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_1^2, (2,1)) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1)$	see fig. ??
12_6^4	EABCDFFDAEBCCFEBAD	$(D_2^2 \times S^1) \bigcup$ `s776`	L10n111
12_7^4	DABCFEFEABDCEFDACB	`o9_44206`	L10n98
12_8^4	DABCFEFDEBACCEAFDB	hyperbolic manifold with Vol = 10.991587130	L10n100
12_9^4	DABCFEFEABDCCDEFAB	$otet10_{0014}$	L10n101
12_{10}^4	FABCDEDEFABCCDEFAB	$otet10_{0028}$	L12n2201
12_{11}^4	DABCFEFDEBACECFADB	hyperbolic manifold with Vol = 10.6669791338	L12n2205
12_{12}^4	CABFDEFCEABDDEACFB	$otet12_{0009}$	L12n2208
12_1^5	EABCDFFEABDCCDFEBA	$(D_2^2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D_2^2 \times S^1)$	L14n63765
12_2^5	DABCFEFEDABCBCFEDA	$otet10_{0027}$	L10n113

Orientable prime 3-manifolds with toric boundary with $c_g = 14$

Name	Code	Manifold	Link
14^2_1	EABCDGFGDFEBCADGEFBAC	$(D^2_1, (3,1))$	L6a3
14^2_2	DABCGEFGFECDBABGDFACE	$(D^2, (2,1), (3,1)) \bigcup_{\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}} (D^2_2 \times S^1)$	see fig. ??
14^2_3	GABCDEFEDGFABCDEFAGCB	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} (D^2_1, (2,1))$	L11n204
14^3_1	EABCDGFGEFCADBCEGAFBD	$(D^2_2, (2,1))$	L12n1998
14^3_2	DABCGEFGFBADCEFCEAGDB	$(M^2_2, (1,0))$	–
14^3_3	DABCGEFFDBECGAEDGCFAB	$(D^2, (2,1), (3,1)) \bigcup_{\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}} (D^2_3 \times S^1)$	see fig. ??
14^3_4	EABCDGFGDFEBCABDGAFEC	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix}1 & 2\\ 1 & 1\end{pmatrix}} (D^2_2 \times S^1)$	L8n6
14^3_5	DABCGEFGEFBDACFGEBACD	$(D^2_1, (3,1)) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} (D^2_2 \times S^1)$	see fig. ??
14^3_6	EABCDGFGFDABECCEFAGDB	$(M^2_1, (1,0)) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} (D^2_2 \times S^1)$	–
14^3_7	EABCDGFGFDABECFDGBACE	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} (D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} (D^2_1, (2,1))$	see fig. ??
14^3_8	DABCGEFGDFCABEBFDECGA	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix},1} M_{L5a1}$	L13n9356
14^3_9	DABCGEFGEFBDACFCGABDE	$\mathrm{t}12066, \mathrm{ooct}02\_00003$	L8n5
14^3_{10}	DABCGEFGDFBACECFAEDGB	$\mathrm{t}12067, \mathrm{ooct}02\_00005$	L6a4

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14^4_1	EABCDGFGFBACEDBCFDGAE	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_2, (2,1))$	see fig. ??
14^4_2	EABCDGFGBEDFACEFAGCDB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (M^2_2, (1,0))$	–
14^4_3	DABCGEFGCFADBEEGABCFD	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_3 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_1, (2,1))$	see fig. ??
14^4_4	EABCDGFGEFCADBBFDGEAC	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_1, (2,1)) \bigcup_{\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}} (D^2_2 \times S^1)$	L11n379
14^4_5	EABCDGFGFECABDCGDAFEB	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}} (D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_2 \times S^1)$	see fig. ??
14^4_6	EABCDGFGDFACEBBFEDGAC	$(M^2_1, (1,0)) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (D^2_2 \times S^1)$	–
14^4_7	EABCDGFGFDEBCAFDEGCAB	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},1} M_{L8n7}$	L14n63157
14^4_8	EABCDGFGFEBCDACGFEBAD	$(D^2_1, (2,1)) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},1} M_{L8n7}$	L14n61549
14^4_9	EABCDGFGDFEBCAFCGADEB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix},1} M_{L6a5}$	L14n62850
14^4_{10}	EABCDGFGFBEACDDCGAFEB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},3} M_{L8n5}$	see fig. ??

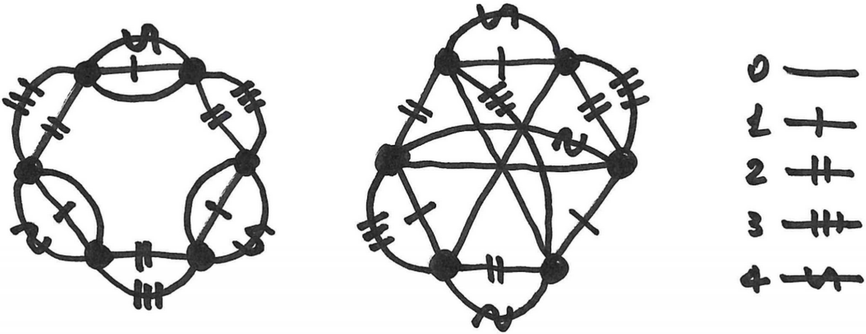
Orientable prime 3-manifolds with toric boundary with $c_g = 14$

Name	Code	Manifold	Link
14^4_{11}	DABCGEFGEFBDACFGCABDE	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix},1} M_{L8n5}$	L14n62541
14^4_{12}	EABCDGFGEFBDACCGAEFDB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix},1} M_{L6a4}$	see fig. ??
14^4_{13}	EABCDGFGEFBDACBGCEFDA	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix},1} M_{L5a1} \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix},2} (D^2_2 \times S^1)$	see fig. ??
14^4_{14}	EABCDGFGDFACEBFCEGBAD	$\mathrm{otet10\_00011}, \mathrm{ocube02\_00044}$	L8a21
14^4_{15}	EABCDGFGFDABECFDEGCAB	Hyperbolic 11 tetrahedra	L14n60227
14^4_{16}	EABCDGFGFEACBDCDFGAEB	Hyperbolic 12 tetrahedra	L10n96
14^4_{17}	DABCGEFGEFBDACCGAFBDE	Hyperbolic 14 tetrahedra	L11n456
14^4_{18}	DABCGEFGEFBDACFGEACDB	Hyperbolic 14 tetrahedra	L14n63000
14^5_1	EABCDFGGFEBADCCDEGFAB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix}} (D^2_3 \times S^1)$	see fig. ??
14^5_2	DABCGEFGFBADCEECFGABD	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}1&0\\0&-1\end{pmatrix},1} M_{L8n7}$	L12n2249
14^5_3	DABCGEFGCFADBECDEGAFB	$(D^2_2 \times S^1) \bigcup_{\begin{pmatrix}0&1\\1&0\end{pmatrix},1} M_{L8n7}$	L14n63769

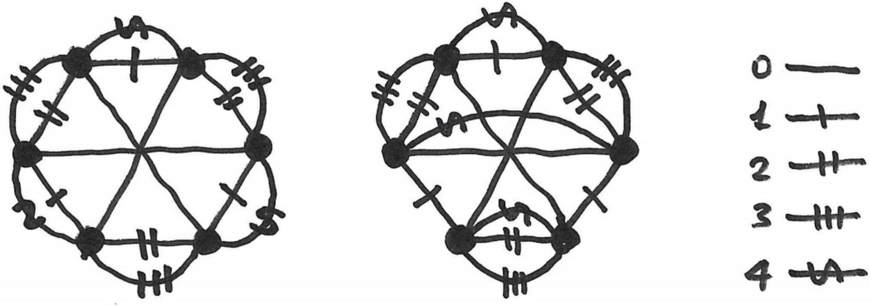
Classification results: $n = 4$

Proposition (Grasselli– Mulazzani, 2018)

Let Γ be a bipartite 5-colored graph of order six. Then M_Γ is one of the following 4-manifolds: S^4, D^4, $S^1 \times D^3$, $S^1 \times S^1 \times D^2$.



Left: $\widehat{M}_\Gamma = M_\Gamma = S^4$
Right: $M_\Gamma = D^4$



Left: $\mathrm{M}_\Gamma = S^1 \times S^1 \times D^2$
Right: $\mathrm{M}_\Gamma = S^1 \times D^3$

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