On a connection between the order of a finite group and the set of conjugacy classes size

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$N(G)$ the set of orders of conjugacy classes of G.

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- $N(Alt_9) = \{1, 105, 112, 210, 1120,$
  $1260, 1344, 1680, 2520, 2880, 3360\}$.

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Definitions

Let p and q be distinct numbers. Say that a group G satisfies the condition $\{p,q\}^*$ and write $G \in \{p,q\}^*$ if we have $\alpha_{\{p,q\}} \in \{|G||_p, |G||_q, |G||_{\{p,q\}}\}$ for every $\alpha \in N(G) \setminus \{1\}$.

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- $Sym_9 \notin \{p,q\}^*$ for any $p,q \in \pi(Sym_9)$.

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A. Beltram and M.J. Filipe, 2006

Let G be a finite soluble group whose conjugacy class sizes are $\{1, n, m, nm\}$, where n and m are coprime positive integers; then G is nilpotent and the integers n and m are prime-power numbers

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Q. Kong and X. Guo, 2010

Let G be a finite group and assume that the conjugacy classes sizes of elements of primary and biprimary orders of G are exactly $\{1, p^a, n, p^a n\}$ with $(p, n) = 1$, where p is a prime and a and n are positive integers. If there is a p-element in G whose index is precisely p^a, then G is nilpotent and $n = q^b$ for some prime $q \neq p$.

Theorem I.G. 2018

If $G \in \{p, q\}^*$ is a group with trivial center, where $p, q \in \pi(G)$ and $p > q > 5$, then $|G|_{\{p,q\}} = |G||_{\{p,q\}}$.

Theorem I.G. 2018

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Corollary

In the hypotheses of the theorem, $C_G(g) \cap C_G(h) = 1$ for every p-element g and every q-element h.