On equitable 2-partitions of the Hamming graphs with eigenvalue λ_2

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joint work with Alexandr Valyuzhenich

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The Hamming graph H(n,q)

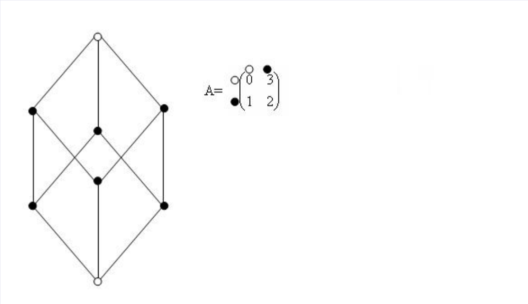
$V = \{(x_1, \ldots, x_n) : x_i \in \{0, \ldots, q-1\}\}$

$(x, y) \in E$ if x and y differ in exactly one position.

Equitable partition

A r-partition $C_1, \ldots, C_r$ of the $V(G)$ is called *equitable* if for any $i, j \in \{1, \ldots, r\}$ a vertex from C_i has exactly A_{ij} neighbors in C_j. The matrix $A = (A_{ij})_{i,j \in \{1,\ldots,r\}}$ is called *the quotient matrix*.

Example: equitable 2-partition of $H(3, 2)$



Spectra of equitable partitions

An eigenvalue of an equitable partition is an eigenvalue of its quotient matrix.

Lloyd's theorem [1,2]*:*

An eigenvalue of an equitable partition of a graph is an eigenvalue of the graph.

[1] Lloyd, S. P. Binary Block Coding, Bell System Tech. J. 36, 517-535 (1957)
[2] D. M. Cvetkovic, M. Doob, H. Sachs, Spectra of graphs, Academic Press, New York, London, (1980)

Eigenvalues of the Hamming graph

The eigenvalues of $H(n, q)$ are
$\lambda_i(n, q) = (q - 1)n - qi, i \in \{0, \ldots, n\}$.

Equitable 2-partitions of H(n,q)

An equitable 2-partition of $H(n,q)$ has two eigenvalues: $\lambda_0(n,q)$ (the valency of the graph) and $\lambda_i(n,q) = A_{11} - A_{21}$, $i \in \{1,\dots,n\}$, where A is the quotient matrix.

Meyerowitz [3]:

Characterization of equitable 2-partitions of $H(n,q)$ with e.v. $\lambda_1(n,q)$.

Problem: characterization of equitable 2-partitions of $H(n,q)$ with e.v. $\lambda_2(n,q)$?

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Equitable 2-partitions of H(n,q): some previous results

Avgustinovich, Heden, Krotov, Phelps, Rifa, Solov'eva, Zinoviev etc: the theory of 1-perfect codes

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Adding/Removing nonessential positions

Let C_1, C_2 be a 2-partition of $H(n,q)$.
$C_i^+ := \{(x_1,\dots,x_n,y) : x \in C_i, y \in \{0,\dots,q-1\}\}$.

Proposition

C_1, C_2 is equitable 2-partition of $H(n,q)$ with e.v. $\lambda_i(n,q)$ iff C_1^+, C_2^+ is equitable 2-partition of $H(n+1,q)$ with e.v. $\lambda_i(n+1,q)$.

An equitable 2-partition C_1, C_2 of $H(n,q)$ is called *reduced* if for any $i \in \{1,\dots,n\}$ there are vertices from C_1 and C_2 that differ only in ith position.

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The only reduced equitable 2-partitions of $H(n,q)$ with e.v. $\lambda_1(n,q)$ are equitable 2-partitions of $H(1,q)$.

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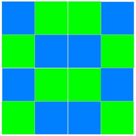
Equitable 2-partitions of $H(1,q) \times H(1,q')$ with e.v. -2

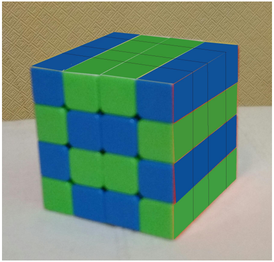
Proposition

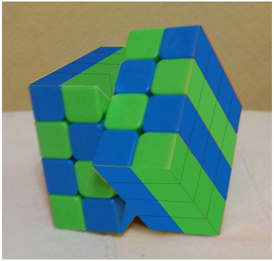
A 2-partition C_1, C_2 of $H(1,q) \times H(1,q')$ is equitable with e.v. -2 iff the percentage of vertices from C_1 in all maximum cliques is the same, i.e. $|K \cap C_1|/|K| = const$ for any maximum clique K.

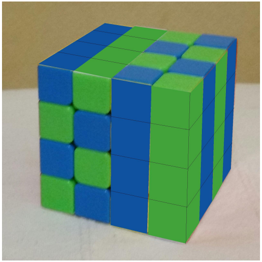
Permutation switchings: idea

Initial equitable 2-partition of H(2,4)









Two equitable partitions of $H(3,4)$ have the same quotient matrix but are nonisomorphic wrt $Aut(H(3,4))$

Permutation switching construction: the initial partition

The initial equitable 2-partition of $H(2, q)$

Induced subgraphs $L_1, \dots, L_{n-1}$, whose vertices part that of $H(1, q)$.

Consider a partition C_1, C_2 of $H(2, q)$:

$|C_1 \cap K|/|K|$ is the same for any maximum clique K of the graphs $L_j \times H(1, q)$, $j \in \{1, \dots, n-1\}$.

Proposition

1. For any j, $C_1 \cap V(L_j \times H(1, q))$, $C_2 \cap V(L_j \times H(1, q))$ is an equitable 2-partition of $L_j \times H(1, q)$ with e.v. -2.
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The initial equitable 2-partition of $H(2,q)$

Induced subgraphs $L_1,\dots,L_{n-1}$, whose vertices part $V(H(1,q))$.
C_1, C_2: percentage of C_1 in the maximum cliques of the subgraph $L_j\times H(1,q)$ is the same for any j.

Add $n-2$ nonessential coordinates

$C_i^+=\{(x_1,x_2,y_3,\dots,y_n):(x_1,x_2)\in C_i, y_j\in\{0,\dots,q-1\}, j\in\{3,\dots,n\}\}$, $i=1,2$

Permutation switching

Let L_j^+ be $L_j\times H(n-1,q)$, $j=1,\dots,n-1$
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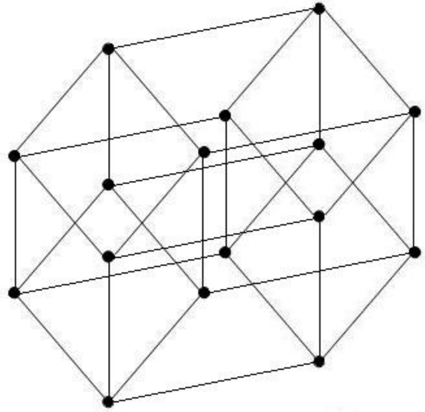

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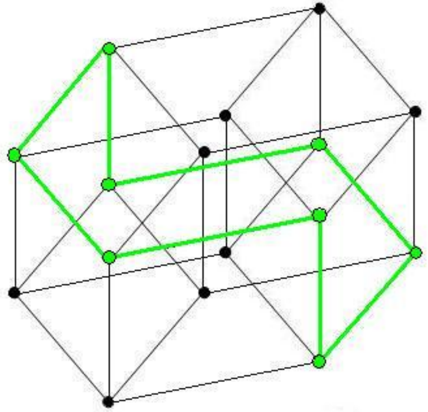

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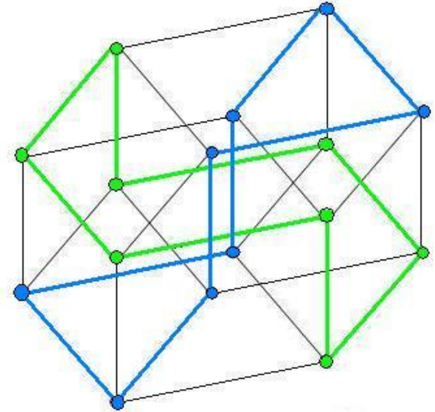

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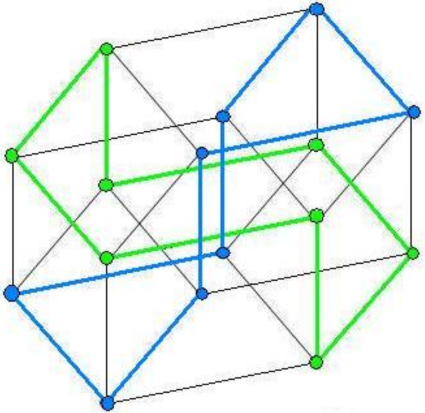

The partition of H(4,2) into 8-cycles is equitable with the quotient matrix $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

Equitable partition from 8-cycles in H(4,2)

Let C_1, C_2 be two induced 8-cycles in $H(4,2)$.

Alphabet liftings: from equitable partition of H(4,2) to that of $H(4,2q')$ (Vorob'ev)

$E_0 = \{0,\dots,q'-1\}$, $E_1 = \{q',\dots,2q'-1\}$.
Consider 2-partition $\overline{C}_1, \overline{C}_2$ of $H(4,2q')$:

$$\overline{C}_i = \bigcup_{(x_1,x_2,x_3,x_4)\in C_i} E_{x_1}\times E_{x_2}\times E_{x_3}\times E_{x_4},$$

$i = 1,2$

Vorob'ev[4]: $\overline{C}_1, \overline{C}_2$ is a reduced equitable 2-partition of $H(4,2q')$ with e.v. $\lambda_2(4,2q')$.

[4] K. V. Vorob'ev, Alphabet lifting construction of equitable partitions of Hamming graphs. *Abstracts of G2R2* (2018).

Equitable partition from 8-cycles in H(4,2)

Let C_1, C_2 be two induced 8-cycles in $H(4,2)$.

Alphabet liftings: from equitable partition of H(4,2) to that of $H(4,2q')$ (Vorob'ev)

$E_0 = \{0,\dots,q'-1\}$, $E_1 = \{q',\dots,2q'-1\}$.

Consider 2-partition $\overline{C}_1, \overline{C}_2$ of $H(4,2q')$:

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Main result

Theorem

The only reduced equitable 2-partitions of $H(n,q)$ with eigenvalue $\lambda_2(n,q)$ are:
1. Reduced equitable partitions of $H(2,q)$ and $H(3,q)$
2. If q is even, the equitable 2-partition of $H(4,q)$ from alphabet liftings of 8-cycles in $H(4,2)$.
3. Equitable 2-partitions of $H(n,q)$ obtained by the permutation switching construction.

Thank you for your attention