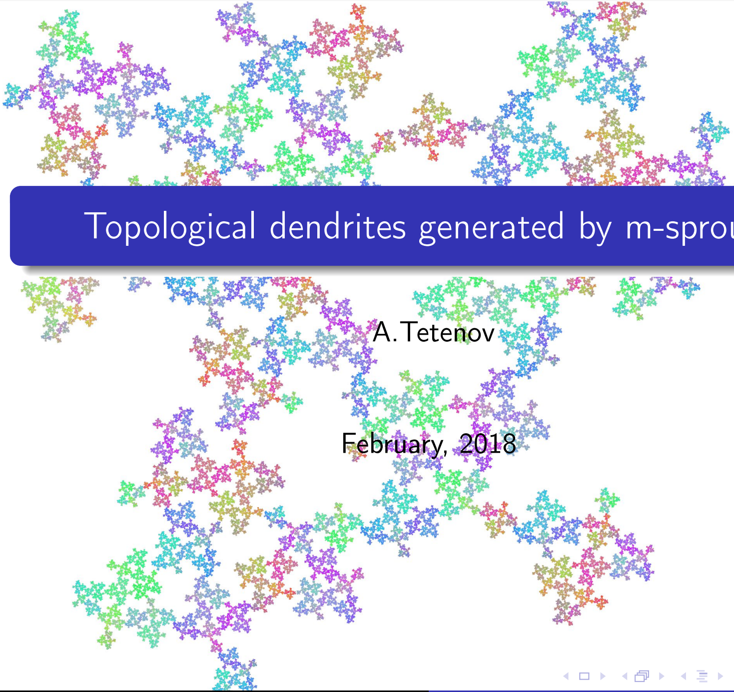


Topological dendrites generated by m-sprouts

A. Tetenov

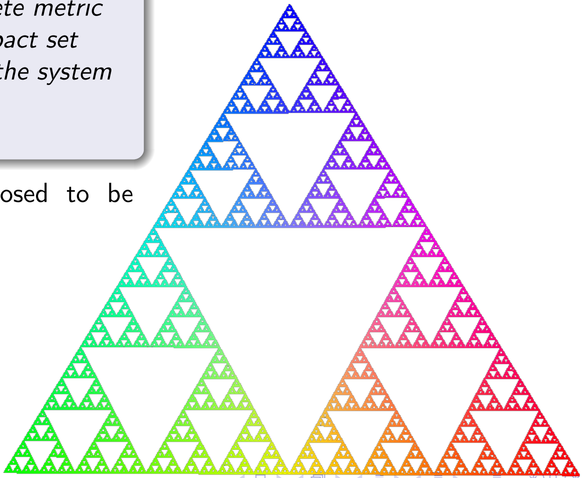
February, 2018

Definition of self-similar sets

Definition

Let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be a system of contraction maps on the complete metric space (X, d). A nonempty compact set $K \subset X$ is called the attractor of the system $\mathcal{S}$, if $K = \bigcup\limits_{i=1}^{m} S_i(K)$.

The maps $S_i \in \mathcal{S}$ are supposed to be similarities and $X = R^2$.



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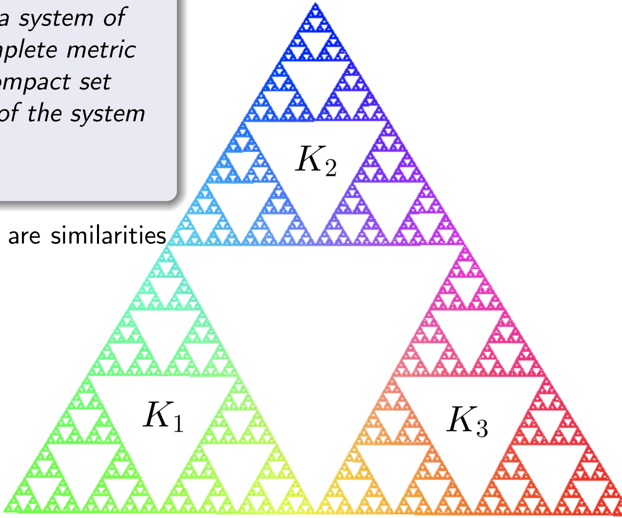

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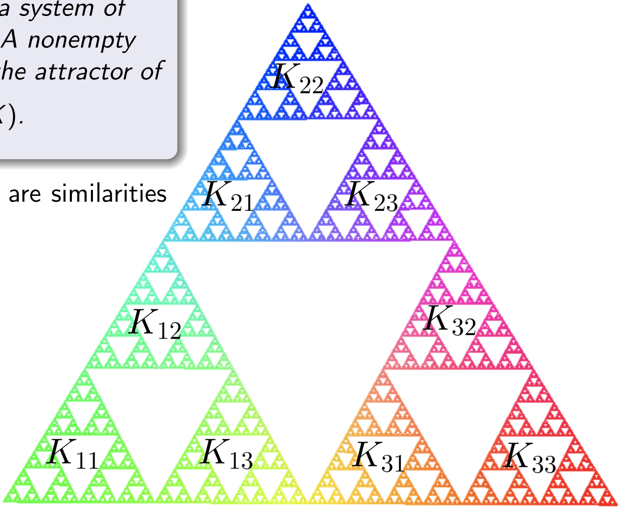
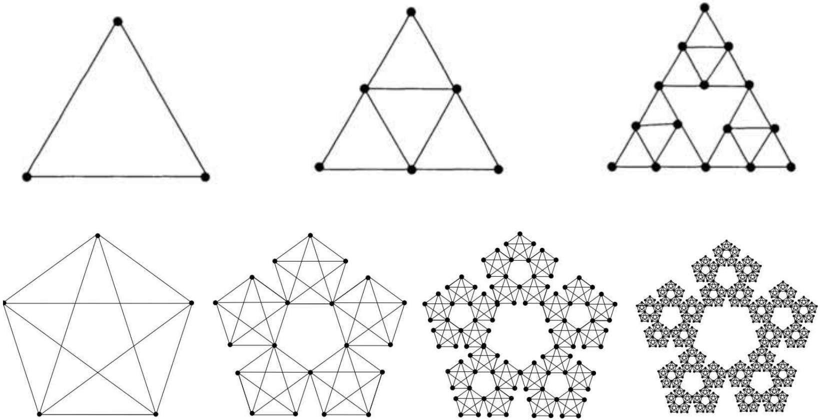
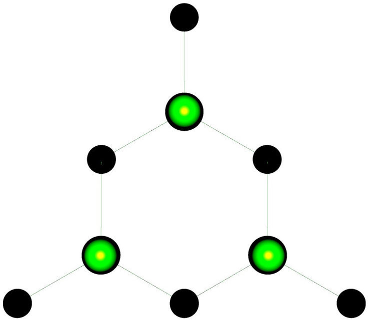
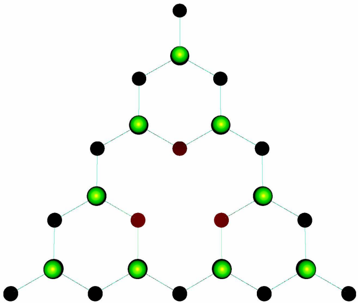
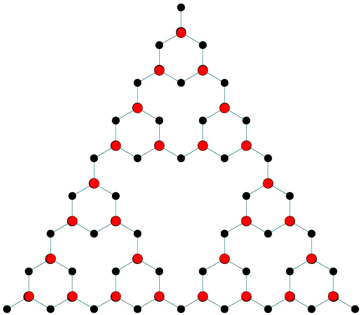
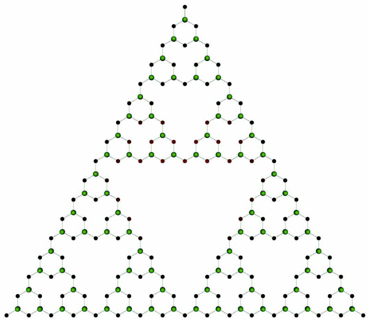










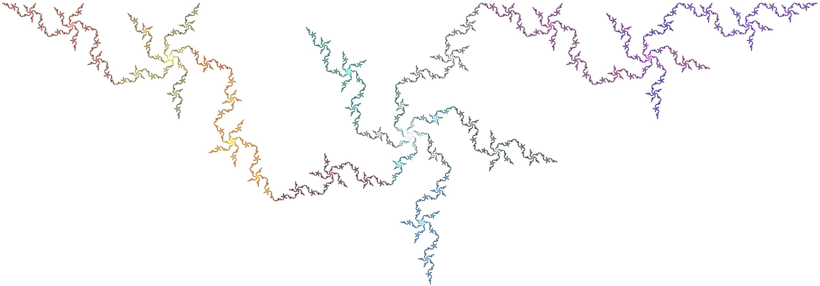


Some pictures of self-similar denrites

Definition

A **dendrite** *is a locally connected continuum containing no simple closed curve.*

The aim of our study are **self-similar dendrites**.



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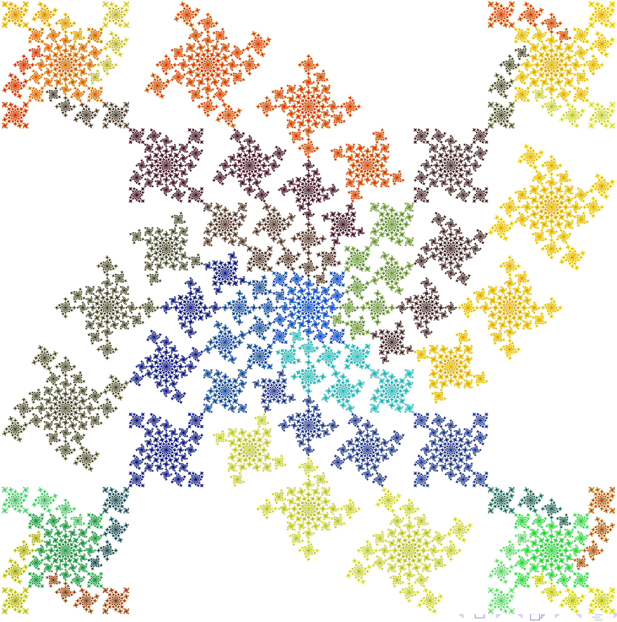

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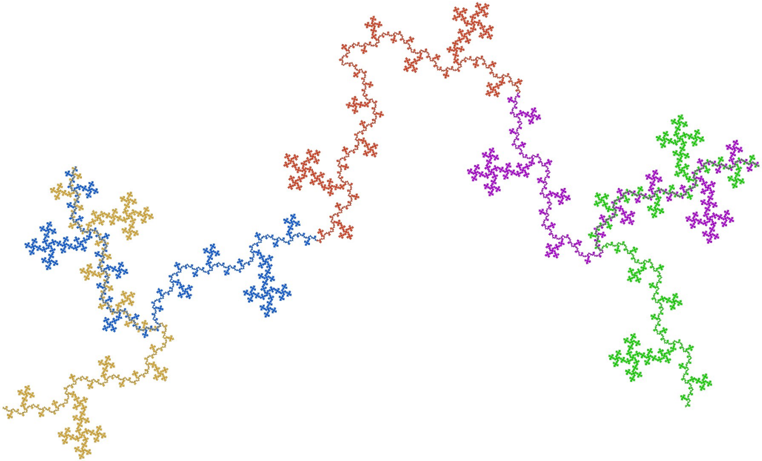
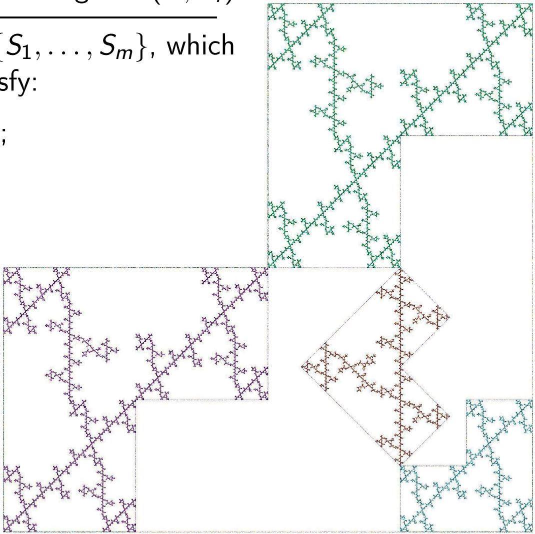


Contractible P-polygonal system of similarities

Let $P \subset \mathbb{R}^2$ be a polygon homeomorphic to a disc
$V_P = \{A_1, ..., A_{n_P}\}$ its vertices with angles $\Omega(P, A_i)$.

Let a system of similarities $\mathcal{S} = \{S_1, \ldots, S_m\}$, which defines polygons $P_i = S_i(P)$ satisfy:

(D1) For any $i \in I$, P_i lies in P;



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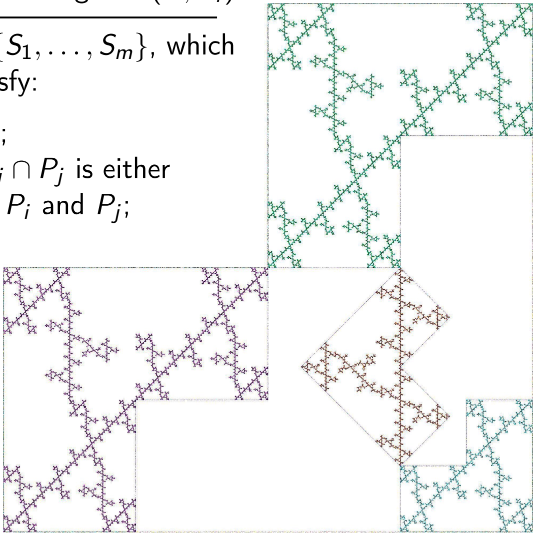
(D1) For any $i \in I$, P_i lies in P;
(D2) For any $i \neq j$, $i, j \in I$, $P_i \cap P_j$ is either empty, or is a common vertex of P_i and P_j;



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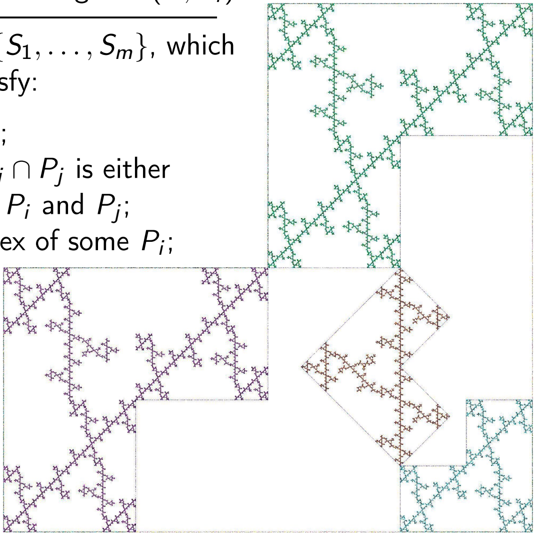
(D2) For any $i \neq j$, $i, j \in I$, $P_i \cap P_j$ is either empty, or is a common vertex of P_i and P_j;
(D3) Each vertex of P is a vertex of some P_i;



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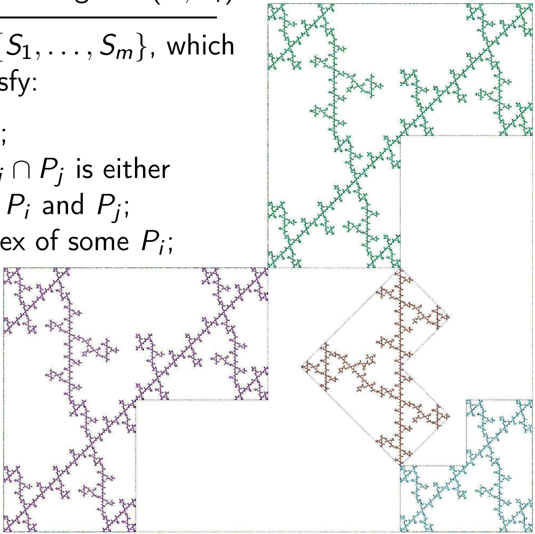
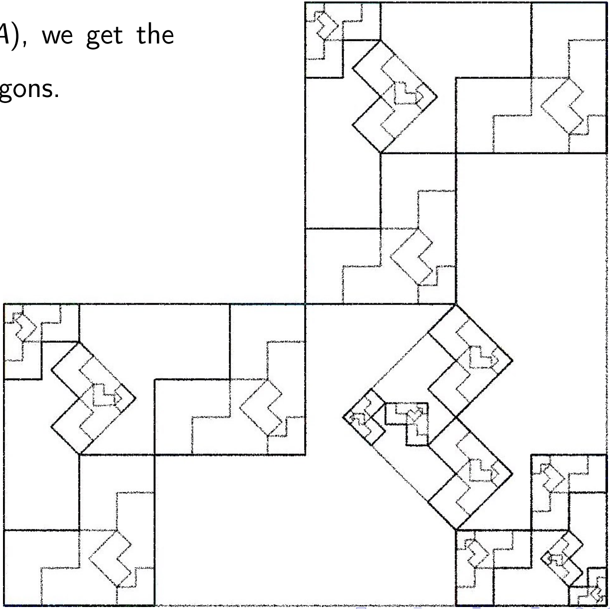
(D3) Each vertex of P is a vertex of some P_i;
(D4) The union $\widetilde{P}$ of all P_i is contractible.

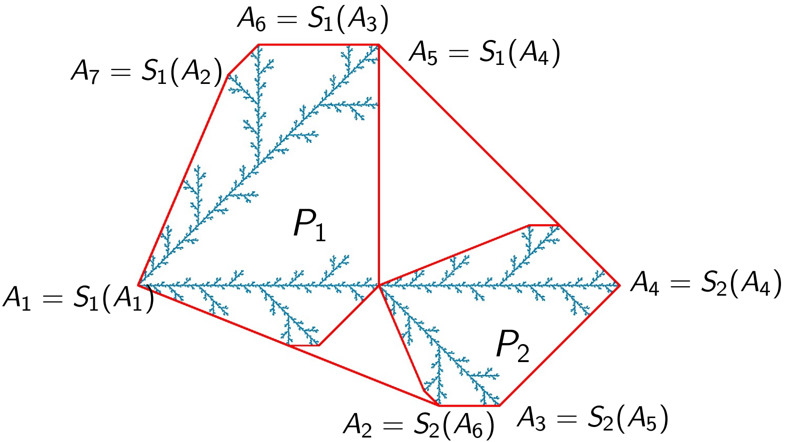


Applying the iterations of Hutchinson's operator $T(A) = \bigcup_{i=1}^{m} S_i(A)$, we get the refining system of subpolygons.



Hata's tree-like set.

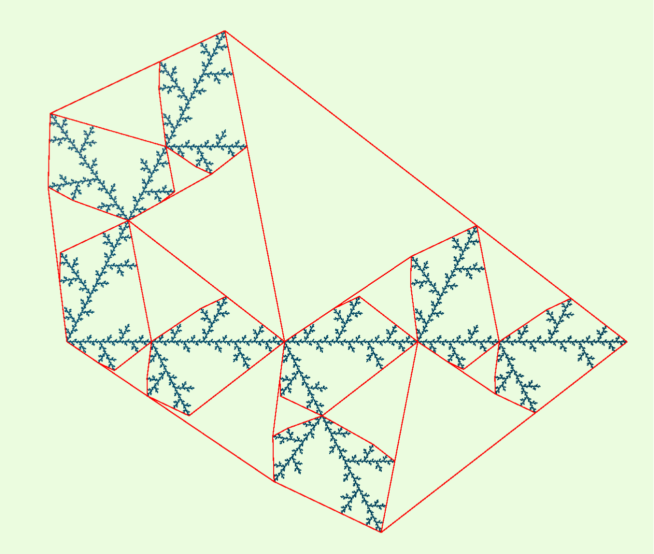
Famous Hata's tree like set is also a polygonal dendrite generated by 7-gons.



Here the maps are $S_1(z) = (1 + i)\bar{z}/2, \quad S_2(z) = (\bar{z} + 1)/2.$

Hata's tree-like set.

Second refinement of the polygonal system for Hata's tree-like set.



;

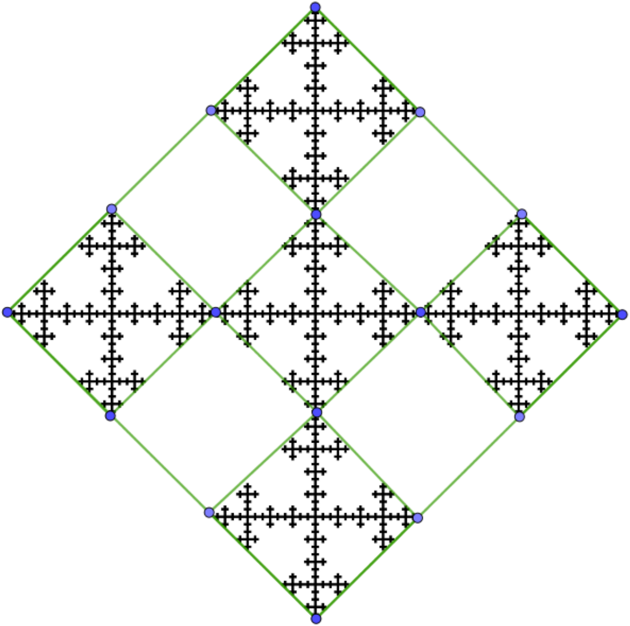
Theorem (Samuel,Tetenov,2016)

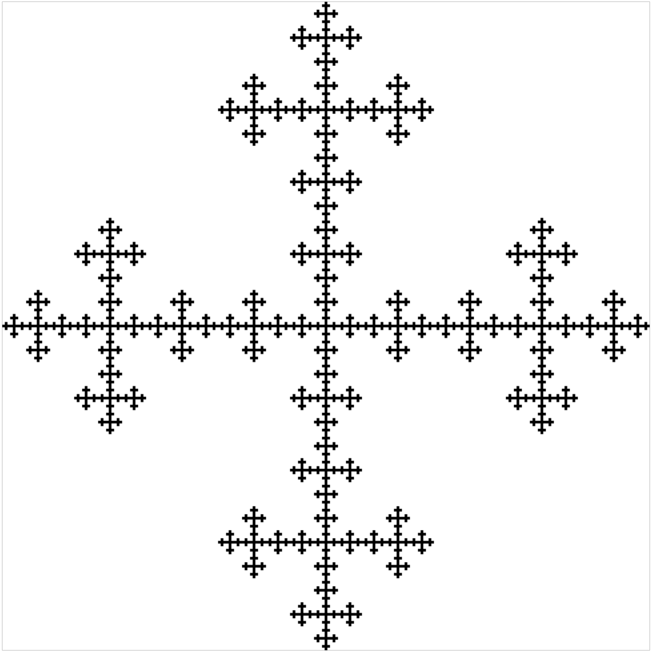
Let $\mathcal{S}$ be a P-polygonal system of similarities, and K its attractor. Then K is a dendrite.

In the dendrite K for any $A_i, A_j \in V_P$ there is unique Jordan arc $\gamma_{ij} \subset K$ connecting A_i, A_j. Connecting them all, we get the main tree of the dendrite K.

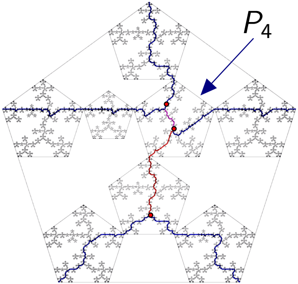
Definition

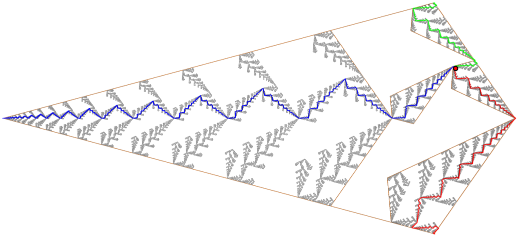
The tree $\hat{\gamma} = \bigcup\limits_{i \neq j} \gamma_{ij}$ *is called* **the main tree** *of the dendrite* K.
The ramification points of $\hat{\gamma}$ *are called* **the main ramification points** *of* K.





In other words, the main tree is a minimal topological tree contained in K which connects all the vertices of the polygon P.





The set of cut points is a countable union of the images of the main tree.

Theorem (Tetenov,Samuel,2016)

(i) $CP(K) \subset G_{\mathcal{S}}(\hat{\gamma})$;

Cut points of K and their order

For each cut point y, which is not an image of some vertex, there is an image $S_{\mathbf{j}}(\hat{\gamma})$ of the main tree for which the order of y in $S_{\mathbf{j}}(\hat{\gamma})$ and in K are the same.

Theorem (Tetenov,Samuel,2016)

(i) $CP(K) \subset G_{\mathcal{S}}(\hat{\gamma})$;
(ii) If $y \notin G_{\mathcal{S}}(V_P)$, *then there are* $\mathbf{j} \in I^*$, $x \in CP(\hat{\gamma})$, *such that* $y = S_{\mathbf{j}}(x)$ *and* $Ord(y, K) = Ord(x, \hat{\gamma}) \leq n_P$.

Theorem (Tetenov,Samuel,2016)

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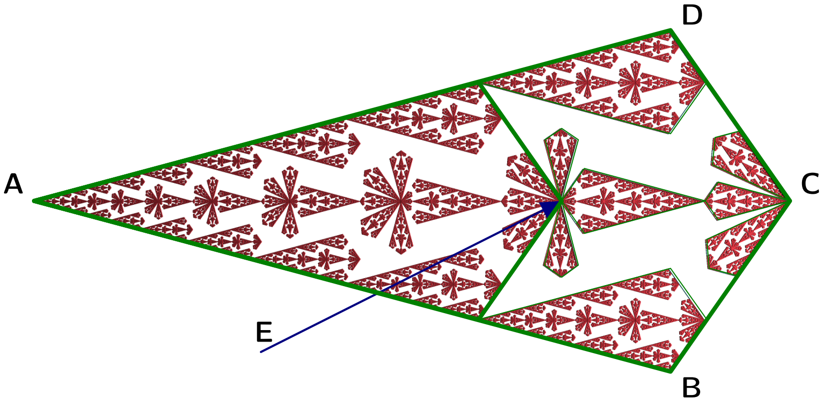
(iii) *If* $y \in G_{\mathbb{S}}(V_P)$, *then*
$Ord(y, K) = \sum_{k=1}^{s} Ord(A'_k, \hat{\gamma}) \leq (n_P - 1)\left(\left\lceil \frac{\theta_F}{\theta_{min}} \right\rceil - 1\right)$, *where* θ_F *is the measure of full angle in* $\mathbb{R}^d$.

Order of cut points:Some examples



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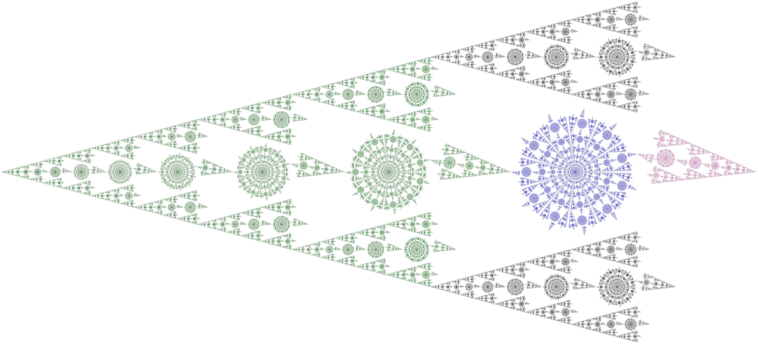


Dimension of cut points and bounded turning

Theorem (Dimensions)

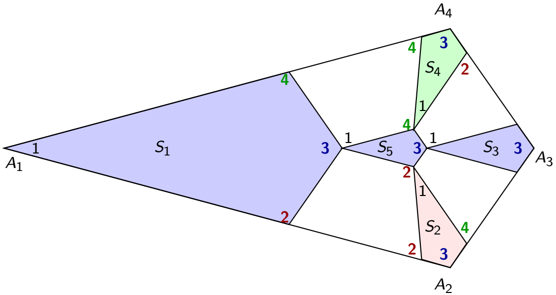
Let $(P, \mathcal{S})$ be a contractible P-polyhedral system and K be its attractor.
(i) $\dim_H(CP(K)) = \dim_H(\hat{\gamma}) \leq \dim_H EP(K) = \dim_H(K)$;
(ii) $\dim_H(CP(K)) = \dim_H(K)$ *iff* K *is a Jordan arc.*

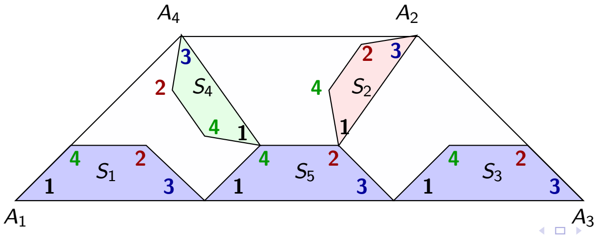
Theorem (Bounded Turning)

For any $x, y \in K$, $\dfrac{\operatorname{diam} \gamma_{xy}}{d(x,y)} \leq \max\left\{ \dfrac{\operatorname{diam} P}{(r \sin(\alpha/2))}, \dfrac{\operatorname{diam} P}{r_1} \right\}$.

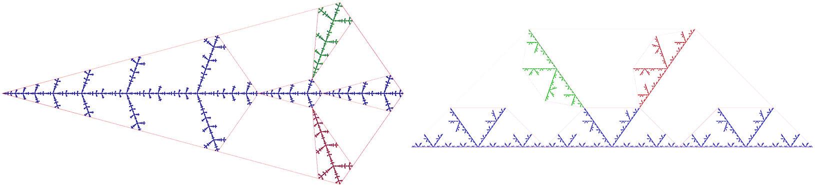
Example of combinatorial equivalence

There are non-homeomorphic polygonal systems, which are combinatorically equivalent

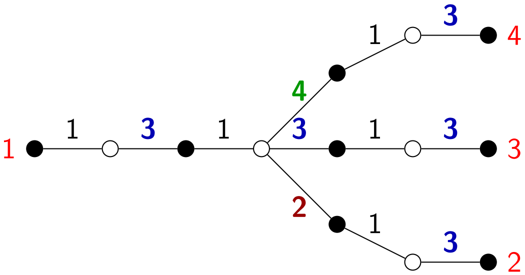




and therefore generate homeomorphic self-similar dendrites:



Their combinatorical structure follows the same pattern:



WE CALL IT m-SPROUT.

Suppose we have:
$I = \{1, ..., m\}$ — the index set;
$\Gamma = (V, E)$ — a tree;

R1: V is divided into 2 parts $V = B \sqcup W$, $E \subset B \times W$ where $\#B \geq m$ and the set of endpoints B_F lies in B.
R2: Injective function $\nu : I \to B$, which assigns indices to some of black vertices,
and the edge coloring function $\varphi : E \to I$, injective on $E(w)$[1]
for any $w \in W$.

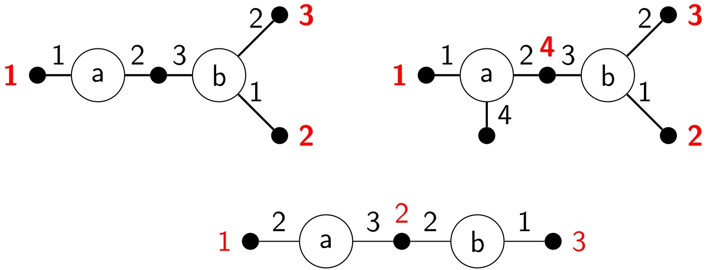
Definition

A tree $\Gamma = \Gamma(B, W, E, \nu, \varphi)$, satisfying **R1,R2**, *is a m-sprout.*

[1] $\Gamma(v) = (N(v), E(v))$ is a neigbourhood of $v \in V$ in Γ.

Definition

A tree $\Gamma = \Gamma(B, W, E, \nu, \varphi)$, *satisfying* **R1,R2**, *is a m-sprout.*

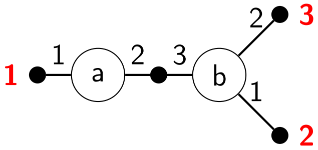


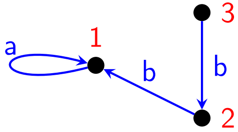
Definition of Index diagram.

Each m-sprout has its INDEX DIAGRAM, built the following way:

The map ν assigns to each $k \in I$ a vertex $b = \nu(k) \in B_I$.
Each $e \in E_b$ has the index $\varphi(e)$ and a vertex $w \in N_1(b)$.
A digraph $\mathcal{G}_I = (I, \mathcal{E})$, has vertices from I, and each edge $\epsilon_e \in \mathcal{E}$ corresponds to some $e \in \bigcup\limits_{b \in \nu(I)} E_b$ so that if $b = \nu(k)$, and $e \in E_b$, then ϵ_e is directed from k to $\varphi(e)$and is marked by $w \in W$, where $w = \omega(e)$.

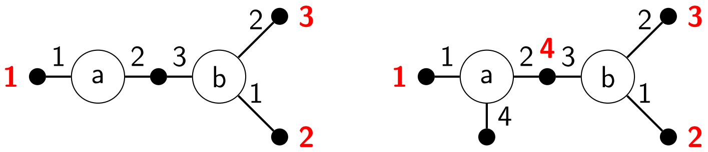
The digraph $\mathcal{G}_I = (I, \mathcal{E})$ is called the index diagram of m-sprout Γ

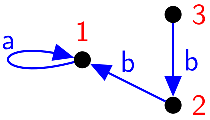




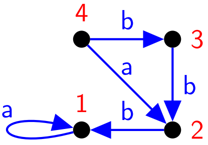
The digraph $\mathcal{G}_I$ may be defined by its INCIDENCE MATRIX.

m-sprouts and their index diagrams





$$\begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ 0 & b & 0 \end{pmatrix}$$



$$\begin{pmatrix} a & 0 & 0 & 0 \\ b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & a & b & 0 \end{pmatrix}$$

Sprouts Γ (top), their index diagrams $\mathcal{G}_I$ and matrices (bottom).

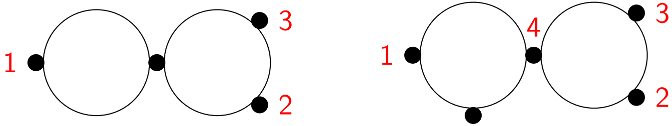
Topological space associated with a sprout

Definition

The space $X = (V, \tau)$, where τ is a topology generated by $\{N(b), b \in B\}$ is a topological space associated with m-sprout Γ.

The properties of topology τ:
(i)All $\{w\}$ are open and $\forall b \in B$, $\{b\}$ is closed.
(ii) All $N(w)$ are closed.
(iii) B and W are discrete subspaces of X.
(iv) Any two $v, v' \in V$ are connected by unique **simple** *bw*-**chain**
(vi) X is connected, and $\forall x \in V \backslash B_F$, $X \backslash \{x\}$ is disconnected.

(vii) X — is a quasicompact connected Kolmogorov space.



Semigroups G_ψ and G_ϕ; $Inv(u)$

For $w \in W$, define[2] $\psi_w : I^0 \to I^0$ by

$$\psi_w(k) = \begin{cases} \varphi(e) \text{ if } \nu(k) \in N(w) \text{ and } e = (w, \nu(k)) \\ 0 \text{ if } \nu(k) \notin N(w) \text{ or } k = 0 \end{cases}$$

Let G_ψ be the semigroup, generated by $\{\psi_w : w \in W\}$

For $w \in W$, define $\phi_w : I \to I$ by:

$\phi_w(k) = \varphi(e)$ if $e \in E(w)$ separates w from $\nu(k)$.

Let G_ϕ be the semigroup, generated by $\{\phi_w : w \in W\}$.

For $u \in G_\psi$ or $u \in G_\phi$, define $Inv(u) = \max\{I' \subset I : u(I') = I'\}$

[2] $I^0 = \{0, 1, ..., m\}$

Composition of m-sprouts

To make the graph refinements, we define an operation of pasting the copies of second sprout into white vertices of first sprout.

Let $\Gamma_1(B_1, W_1, E_1, \nu_1, \varphi_1)$, $\Gamma_2(B_2, W_2, E_2, \nu_2, \varphi_2)$ be m-sprouts. Define the composition $\Gamma(B, W, E, \nu, \varphi) = \Gamma_1 * \Gamma_2$ by :

$W = W_1 \times W_2$, $E = W_1 \times E_2$

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$W = W_1 \times W_2$, $E = W_1 \times E_2$ and $B = (W_1 \times B_2 \cup B_1)/R$, where the equivalence R is generated by identifications

$$w_1 b_2 \sim b_1, \text{ iff } \quad b_2 \in \nu(I) \text{ and } \nu(\varphi(b_1 w_1)) = b_2$$

Define $\nu(k) = \widetilde{\nu_1(k)}$ and $\varphi(b_1 e_2) = \varphi_2(e_2)$.

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Proposition

$\Gamma = \Gamma_1 * \Gamma_2$ *is an m-sprout.*

Lemma

Composition $\Gamma_1 * \Gamma_2$ *is associative.*

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$W = W_1 \times W_2$, $E = W_1 \times E_2$ and $B = (W_1 \times B_2 \cup B_1)/R$, where the equivalence R is generated by identifications

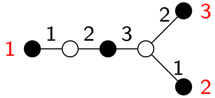
$$w_1 b_2 \sim b_1, \text{ iff } \quad b_2 \in \nu(I) \text{ and } \nu(\varphi(b_1 w_1)) = b_2$$

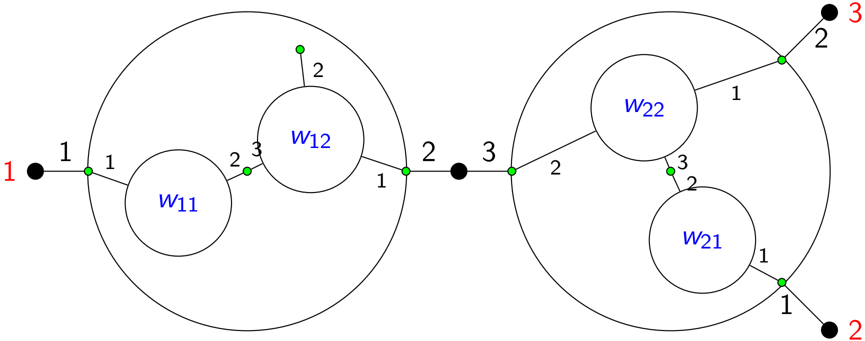
Define $\nu(k) = \widetilde{\nu_1(k)}$ and $\varphi(b_1 e_2) = \varphi_2(e_2)$.

Theorem

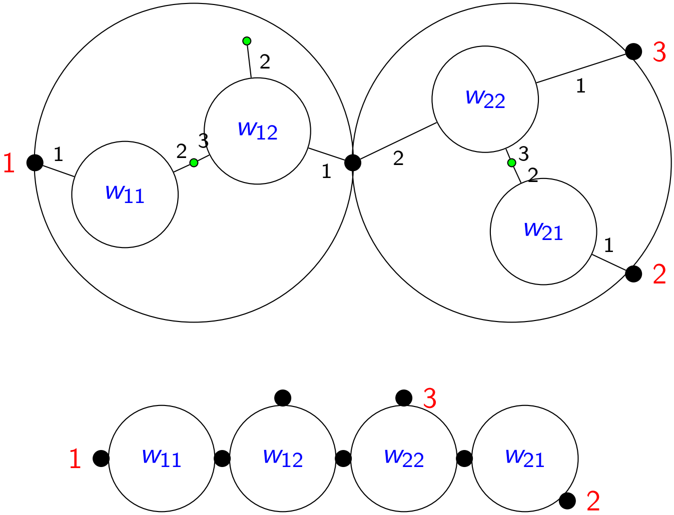
m-sprouts form a semigroup with m-pod as the unity.

Look how we paste the sprout into itelf:





Composition of m-sprouts: step 2



Each $w \in W_1$ defines an embedding of Γ_2 to Γ.

For each $w_1 \in W_1$ define $f_{w_1} by : \Gamma_2 \to \Gamma$:
$f_{w_1}(b_2) = \widetilde{w_1 b_2}$, $f_{w_1}(w_2) = w_1 w_2$, $f_{w_1}(e_2) = w_1 e_2$.

1. $f_{w_1} : \Gamma_2 \to \Gamma$ is an isomorphic embedding,
2. $\Gamma = \bigcup\limits_{w \in W_1} f_w(\Gamma_2)$
3. If $w, w' \in W_1, w \neq w'$, $f_w(\Gamma_2) \cap f_{w'}(\Gamma_2) = N(w) \cap N(w')$.

We also define the embedding J of B_1 to B by $J(b_1) = \widetilde{b_1}$ [3]

[3] $\widetilde{b_1}$ is a class of b_1 with respect to R

The maps f_w for X

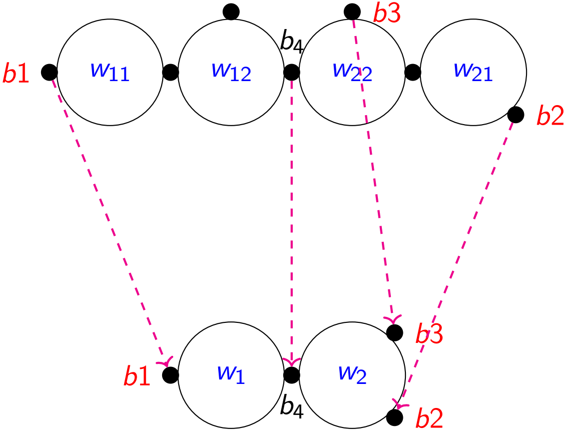
Consider the restrictions of f_w to topological space X_1

1.$f_{w_1} : X_2 \to X$ are homeomorphic embeddings, and
2. $X = \bigcup\limits_{w \in W_1} f_w(X_2)$,
3. if $w, w' \in W_1, w \neq w'$, then $f_w(X_2) \cap f_{w'}(X_2) = N(w) \cap N(w')$.

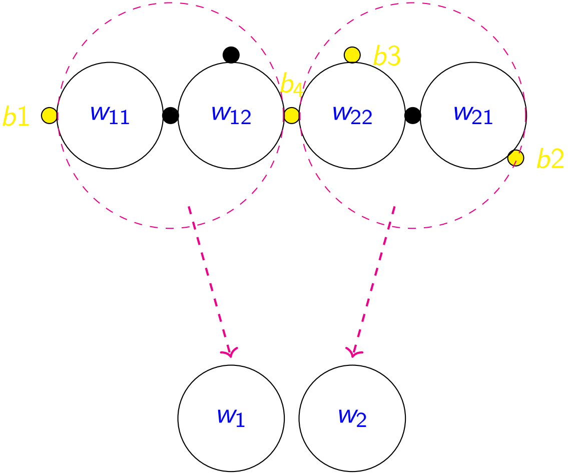
We define the natural projection $\pi : X \to X_1$ by:
If $J(b_1) = \widetilde{b_1}$ and $\pi(x) = \begin{cases} w_1, & \text{if} \quad x \in f_{w_1}(X_2) \setminus J(B_1); \\ b_1, & \text{if} \quad x = J(b_1). \end{cases}$

Then $\pi \circ J = Id|_{B_1}$

Black points from $J(B_1)$ are identically mapped to themselves,



While all the other points go to respective w_1-s.



Diagrams for triple product

We need the following commutative diagrams for topological spaces associated with different products of Γ_1, Γ_2 and Γ_3:

Let $\Gamma_1, \Gamma_2, \Gamma_3$ be m-sprouts, denote $\Gamma_i * \Gamma_j = \Gamma_{ij}$, $\Gamma_1 * \Gamma_2 * \Gamma_3 = \Gamma_{123}$.
Let X_{ij}, X_{123} be resp. topology spaces,
let $\pi_{\mathbf{i}}^{\mathbf{j}}$ for $\mathbf{i} \sqsubset \mathbf{j}$ be a projection of $X_{\mathbf{j}}$ to $X_{\mathbf{i}}$, and
$J_{\mathbf{j}}^{\mathbf{i}}$ be the embedding of $B_{\mathbf{i}}$ to $B_{\mathbf{j}}$.
Let $w \in W_1$. Consider $f_w : X_2 \to X_{12}$ and $f_w : X_{23} \to X_{123}$. Then the diagrams:

$$\begin{tikzcd} X_2 \arrow[d, "f_w"] & X_{23} \arrow[l, "\pi_2^{23}"'] \arrow[d, "f_w"] \\ X_{12} & X_{123} \arrow[l, "\pi_{12}^{123}"'] \end{tikzcd} \qquad \begin{tikzcd} B_2 \arrow[r, "J_{23}^2"] \arrow[d, "f_w"] & B_{23} \arrow[d, "f_w"] \\ B_{12} \arrow[r, "J_{123}^{12}"] & B_{123} \end{tikzcd}$$

are commutative

Sequences Γ^n and X_n

Let $\Gamma = \Gamma(B, W, E, \nu, \varphi)$ be a m-sprout. Define $\Gamma^n = \underbrace{\Gamma * \Gamma * ... * \Gamma}_{\text{n times}}$

Let $X_1, X_2, ..., X_n, ...$ be respective topological spaces, so $X_n = B_n \sqcup W_n$.
For all parts of the entries of the sequence $X_1, X_2, ..., X_n, ...$ we define f_w, J and π maps:

$$\pi_{n,k} : X_{n+k} \to X_n, \quad J_{n,k} : B_n \to B_{n+k}, \quad f_w : X_n \to X_{n+1}$$

$$\forall n \in \mathbb{N},\ w \in W,\ X_{n+1} = \bigcup_{w \in W} f_w(X_n)$$

for $w, w' \in W, w \neq w', f_w(X_n) \cap f_{w'}(X_n) = N(w) \cap N(w')$

Diagrams for X_n

The following diagrams are commutative:

$$
\begin{tikzcd}
X_1 & X_2 \arrow[l, "\pi_{1,1}"'] \arrow[d, "f_w"] & X_3 \arrow[l, "\pi_{2,1}"'] \arrow[d, "f_w"] & \cdots\cdots \arrow[l, "\pi_{3,1}"'] & X_n \arrow[l, "\pi_{n-1,1}"'] \arrow[d, "f_w"] & \ldots \arrow[l, "\pi_{n,1}"'] \\
X_2 & X_3 \arrow[l, "\pi_{2,1}"'] & X_4 \arrow[l, "\pi_{3,1}"'] & \cdots\cdots \arrow[l, "\pi_{4,1}"'] & X_{n+1} \arrow[l, "\pi_{n,1}"'] & \ldots \arrow[l, "\pi_{n+1,1}"']
\arrow[from=1-1, to=2-1, "f_w"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
X_1 & X_2 \arrow[l, "\pi_{2,1}"'] & X_3 \arrow[l, "\pi_{2,1}"'] & \cdots\cdots \arrow[l, "\pi_{3,1}"'] & X_n \arrow[l, "\pi_{n-1,1}"'] & \ldots \arrow[l, "\pi_{n,1}"'] \\
B_1 \arrow[u, "\cup"] \arrow[r, "J_{1,1}"] & B_2 \arrow[u, "\cup"] \arrow[r, "J_{2,1}"] & B_3 \arrow[u, "\cup"] \arrow[r, "J_{3,1}"] & \cdots\cdots \arrow[r, "J_{n-1,1}"] & B_n \arrow[u, "\cup"] \arrow[r, "J_{n,1}"] & \ldots
\end{tikzcd}
$$

Let $X = \varprojlim X_n$.
Then the maps $f_w : X \to X$, $J_n : B_n \to X$ and $\pi_n : X \to X_n$ are well-defined so that:

1. $f_{w_1} : X \to X$ are homeomorphic embeddings, and $X = \bigcup\limits_{w \in W_1} f_w(X)$,
2. if $w, w' \in W_1, w \neq w'$, then $f_w(X) \cap f_{w'}(X) = N^1(w) \cap N^1(w')$.
3. The following diagrams are commutative

$$\begin{tikzcd} X_n \arrow[d, "f_w"] & X \arrow[l, "\pi_n"'] \arrow[d, "f_w"] \\ X_{n+1} & X \arrow[l, "\pi_{n+1}"'] \end{tikzcd} \qquad \begin{tikzcd} X_n & X \arrow[l, "\pi_n"] \\ B_n \arrow[u, "\cup"] \arrow[r, "J_n"] & B \arrow[u, "\cup"'] \end{tikzcd}$$

The simple chains $C_n(x, y)$ and chain equations.

1.For any $x, y \in X$ and $n \in \mathbb{N}$, there is unique simple bw-chain $C_n(x, y) \subset X_n$, connecting $\pi_n(x)$ and $\pi_n(y)$.

---

2. For any $n, k \in \mathbb{N}$, $\qquad \pi_{n,k}(C_{n+k}(x, y)) = C_n(x, y)$

---

3. For any $x, y \in X$ there is a sequence of continuous functions $f_n : [0, 1] \to C_n(x, y)$ such that $\pi_{n,k} \circ f_{n+k} = f_n$

---

4. For any $b, b' \in \nu(I)$ there are unique s and s-tuples $j_1, ..., j_s$, $k_1, ..., k_s$ and $l_1, ..., l_s$ so that for any n, we have the CHAIN EQUATIONS:

$$C_{n+1}(b, b') = \bigcup_{i=1}^{s} f_{w_{j_i}}(C_n(b_{k_i}, b_{l_i}))$$

The semigroup G_ψ defines whether X is Hausdorff.

Theorem

X is a Hausdorff space IFF for any $u \in G_\psi$, $\#Inv(u) \leq 1$.

Lemma

If $\#Inv(u) \leq 1$, for any x, y there is unique simple path $\gamma_{x,y} : [0, 1] \to X$ with endpoints x and y.

X is a dendrite.

Theorem

X is a dendrite IFF for any $u \in G_\psi$, $\#Inv(u) \le 1$.

Proposition

For any $b, b' \in \nu(I)$ we have the MAIN TREE EQUATIONS:

$$\gamma_{bb'} = \bigcup_{i=1}^{s} f_{w_{j_i}}(\gamma_{b_{k_i} b_{l_i}})$$

X has finite ramification order.

Theorem (Tetenov,2017)

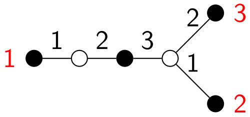
If the index diagram of Γ does not contain cyclic vertices with outgoing ramification order ≥ 2, then the ramification order for the point of X is bounded.

Proposition

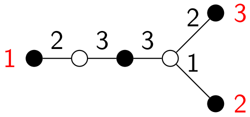
For any $w_{\mathbf{j}} = w_{j_1 j_2 \ldots j_k}$, the fixed point of $f_{w_{\mathbf{j}}}$ has ramification order in X is equal to $\#Inv(\phi_{w_{\mathbf{j}}})$.

8 possible types of 3-sprouts with $\#W = 2$

A1.



A5.



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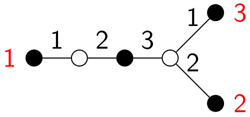

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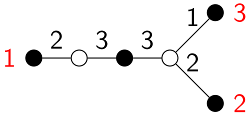

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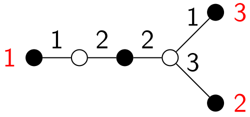
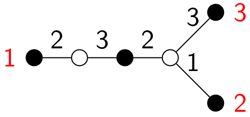
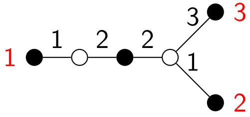


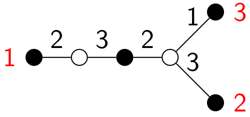
A7.



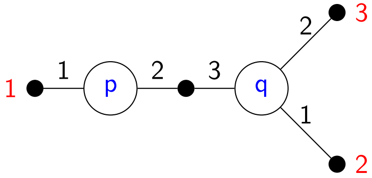
A4.



A8.



Type A1



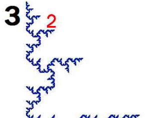
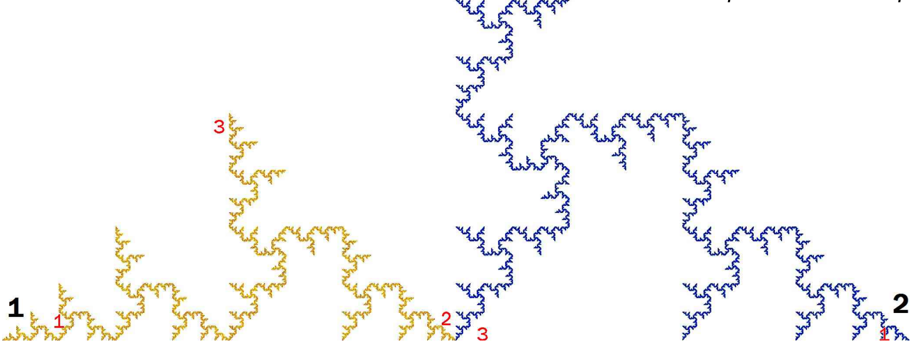
$d_{13} = c;\ d_{12} = 1;\ d_{23} = q$

S.p.metrics equations:

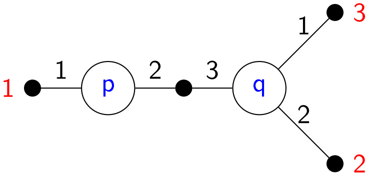
$$p + qc = 1$$

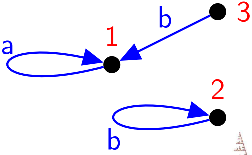
$$p + q^2 = c$$

$$p = \frac{1 - q^3}{1 + q}; \qquad c = \frac{1 + q^2}{1 + q}$$

Type A2



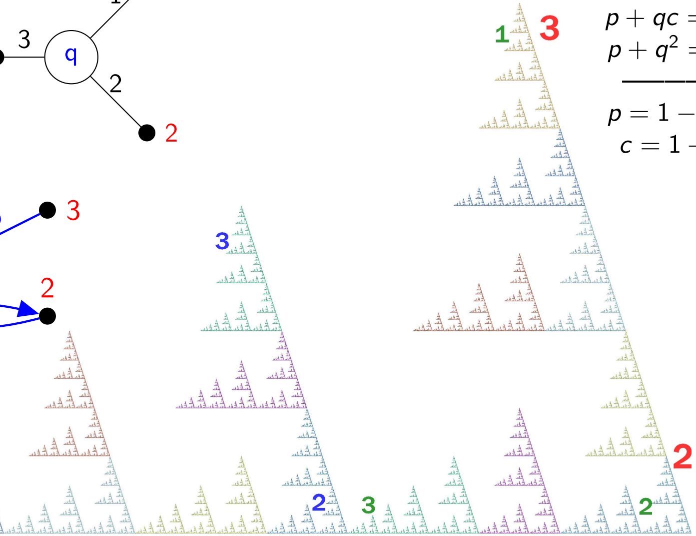


$d_{13} = c;\ d_{12} = 1;\ d_{23} = q$

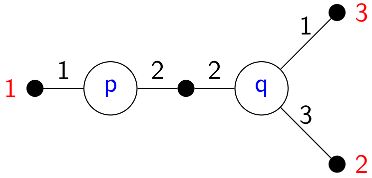
S.p.metrics equations:

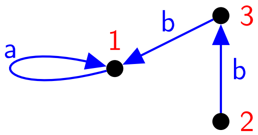
$$p + qc = c$$
$$p + q^2 = 1$$

$$p = 1 - q^2$$
$$c = 1 + q$$



Type A3



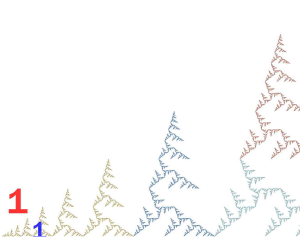
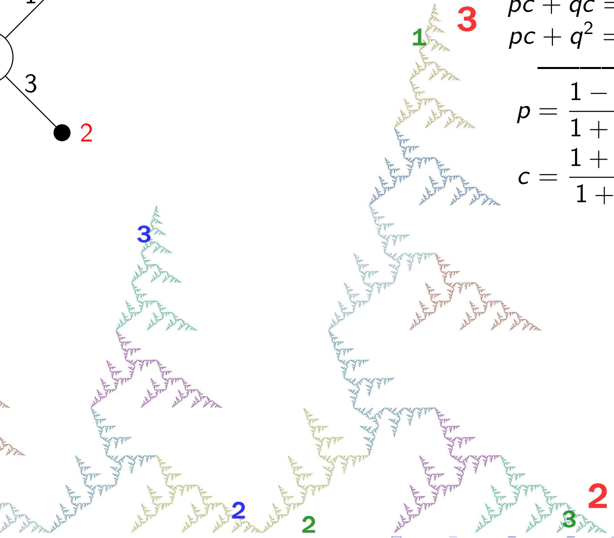


$d_{13} = 1;\ d_{12} = c;\ d_{23} = q$

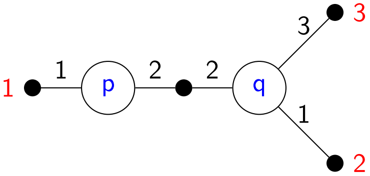
S.p.metrics equations:

$$pc + qc = 1$$
$$pc + q^2 = c$$

$$p = \frac{1 - q^3}{1 + q^2}$$
$$c = \frac{1 + q^2}{1 + q}$$

Type A4 — a Jordan zipper

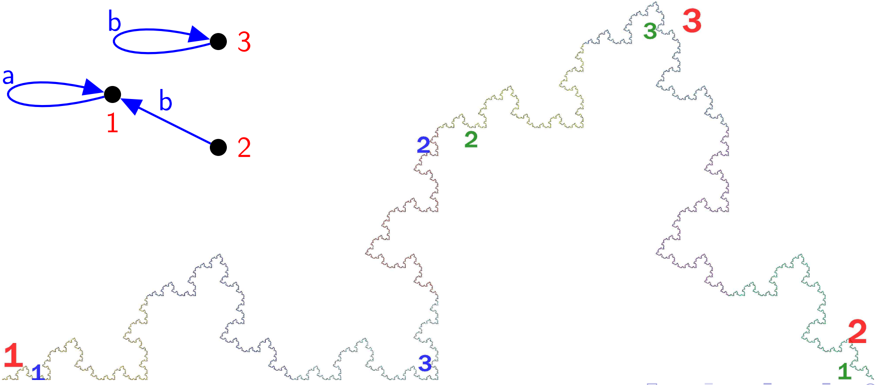


$d_{12} = 1$; $d_{13} = c$; $d_{23} = qc$

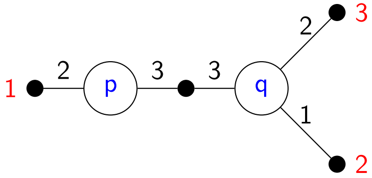
S.p.metrics equations:

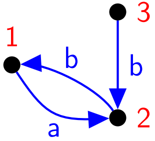
$$p + q = 1$$
$$p + q^2 c = c$$

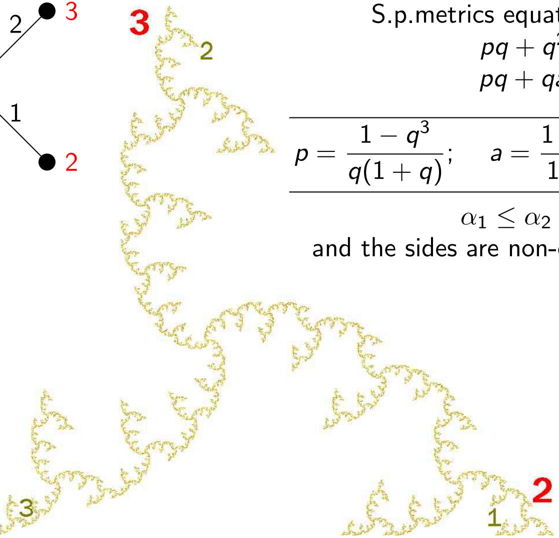
$$p = 1 - q$$
$$c = 1/(1 + q)$$



Type A5, self-affine only







$d_{13} = a;\ d_{12} = 1;\ d_{23} = q$

S.p.metrics equations:

$$pq + q^2 = a$$

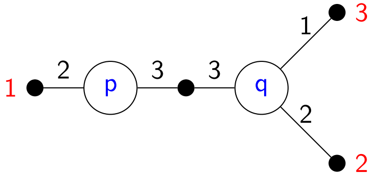
$$pq + qa = 1$$

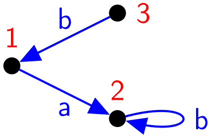
$$p = \frac{1-q^3}{q(1+q)}; \qquad a = \frac{1+q^2}{1+q}$$

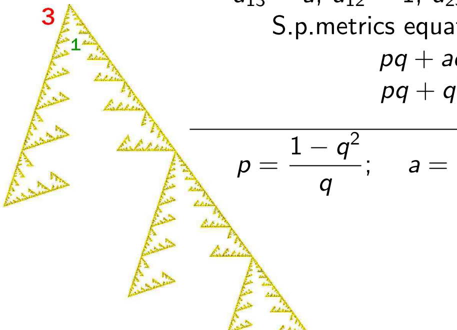
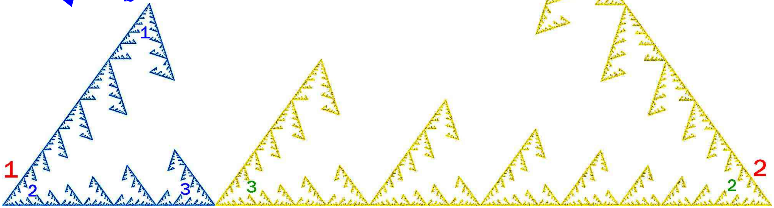
$\alpha_1 \leq \alpha_2 \leq \alpha_1$

and the sides are non-equal

Type A6





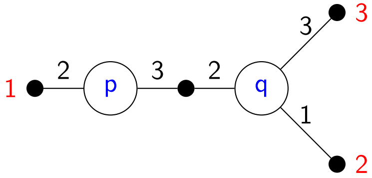
$d_{13} = a;\ d_{12} = 1;\ d_{23} = q$

S.p.metrics equations:

$$pq + aq = a$$
$$pq + q^2 = 1$$

$$p = \frac{1 - q^2}{q}; \qquad a = 1 + q$$

Type A7- self-affine only



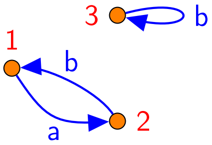
$d_{13} = 1;\ d_{12} = a;\ d_{23} = q$

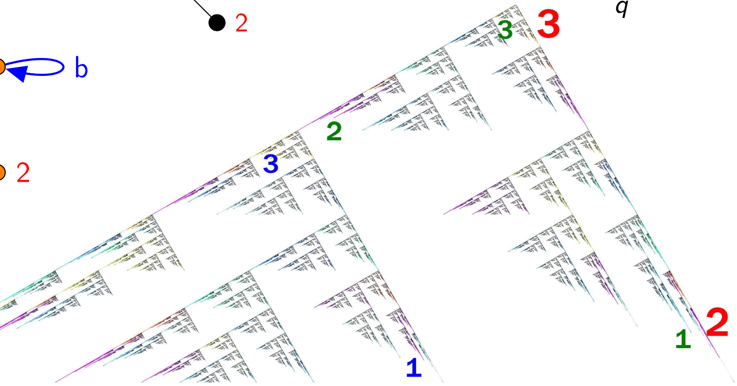
S.p.metrics equations:

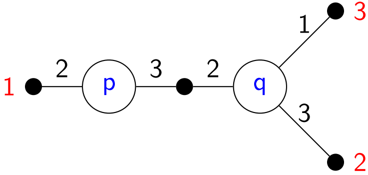
$$pq + aq = a$$

$$pq + q^2 = 1$$

$$p = \frac{1 - q^2}{q};\ a = 1 + q$$





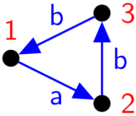


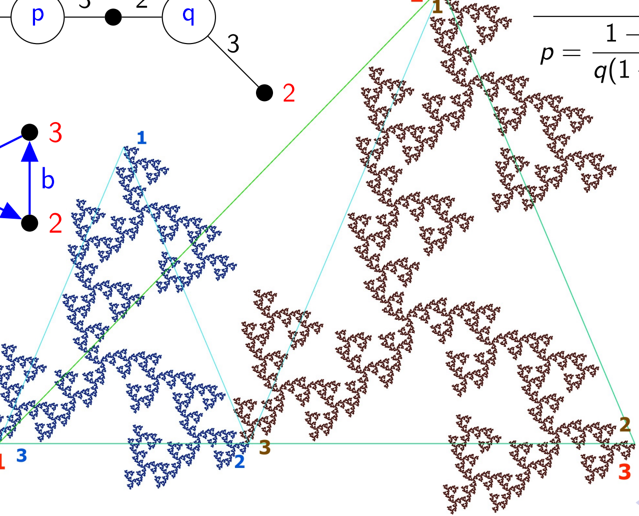
$d_{13} = 1$; $d_{12} = a$; $d_{23} = q$

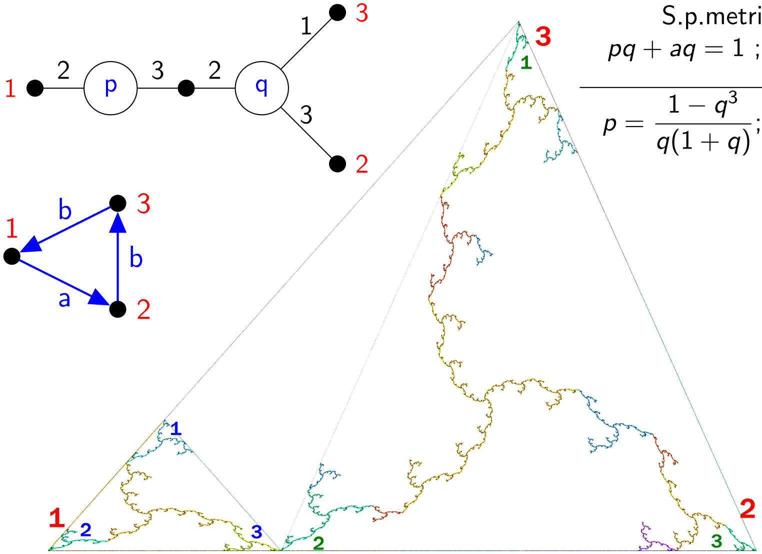
S.p.metrics equations:

$pq + aq = 1$; $pq + q^2 = a$

$$p = \frac{1 - q^3}{q(1+q)}; \quad a = \frac{1 + q^2}{1 + q}$$







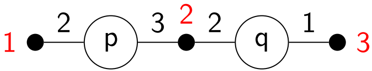
$d_{13} = 1$; $d_{12} = a$; $d_{23} = q$

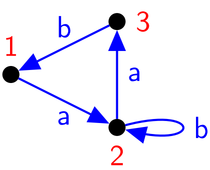
S.p.metrics equations:

$pq + aq = 1$; $pq + q^2 = a$

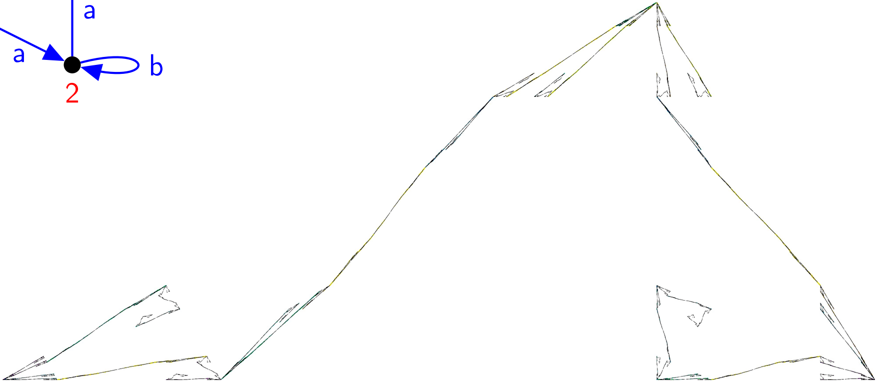
$$p = \frac{1 - q^3}{q(1+q)}; \quad a = \frac{1+q^2}{1+q}$$

Infinitely ramified self-affine dendrite





$d_{12} = pd_{23}$; $d_{23} = qd_{12}$,
so there is no self-similar
short path metrics on K
b_2 is cyclic point with outgoing order
2 in $\mathcal{G}_I$, so b_2 has infinite order in K



TO BE CONTINUED