Structure and Automorphism group of Involution G-Graphs and Cayley Graphs

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Let G be a finite group with identity element 1 and Inv be the set of all involutions in G. The involution $G-$graph $\phi = \phi(G, Inv)$ is a particular $G-$graph introduced by Alain Bretto 2005. The set of vertices of this graph contains the cycles of $(s)x = (x, sx, s^2x, \cdots, s^{o(s)-1}x)$, where $s \in S$ and $x \in T_s$, a set of right transversal of s in G. Two vertices $(s_1)x$ and $(s_2)y$ are adjacent in ϕ when the intersection set of their supports has more than one element. The $G-$graphs have many properties similar to Cayley graphs but they are more general than the Cayley graphs and most of the well-known graphs are $G-$graphs. The aim of this work is obtaining the structures and the automorphism groups of the involution $G-$graphs for some classes of finite groups and some simple groups generated by their involutions and then comparing them with the automorphism groups of the involution Cayley graphs. To do our computational work we use GAP and the finite representation of the groups. Some of the finite groups which we will consider have the representations as follows:

$$D_{2n} = \langle a, b | a^n = b^2 = 1, b^{-1}ab = a^{-1} \rangle,$$
$$V_{8n} = \langle a, b | a^{2n} = b^4 = 1, aba = b^{-1}, ab^{-1}a = b \rangle,$$
$$SD_{8n} = \langle a, b | a^{4n} = b^2 = 1, bab = a^{2n-1} \rangle,$$
$$T_{4n} = \langle a, b | a^{2n} = 1, a^n = b^2, b^{-1}ab = a^{-1} \rangle,$$
$$U_{2nm} = \langle a, b | a^{2n} = b^m = 1, aba^{-1} = b^{-1} \rangle.$$

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