## Intersection of conjugate solvable subgroups in classical groups of Lie type

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Assume that a finite group $G$ acts on a set $\Omega$. An element $x \in \Omega$ is called a *regular point* if $|xG| = |G|$, i.e. if the stabilizer of $x$ is trivial. Define the action of the group $G$ on $\Omega^k$ by the rule

$$g : (i_1, \ldots, i_k) \mapsto (i_1 g, \ldots, i_k g).$$

If $G$ acts faithfully and transitively on $\Omega$, then the minimal number $k$ such that the set $\Omega^k$ contains a $G$-regular point is called the *base size* of $G$ and is denoted by $b(G)$. For a positive integer $m$ the number of $G$-regular orbits on $\Omega^m$ is denoted by $Reg(G, m)$ (this number equals 0 if $m < b(G)$). If $H$ is a subgroup of $G$ and $G$ acts by the right multiplication on the set $\Omega$ of right cosets of $H$ then $G/H_G$ acts faithfully and transitively on the set $\Omega$. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $b(G/H_G)$ and $Reg(G/H_G, m)$ by $b_H(G)$ and $Reg_H(G, m)$ respectively.

Thus $b_H(G)$ is the minimal number $k$ such that there exist elements $x_1, \ldots, x_k \in G$ for which $H^{x_1} \cap \ldots \cap H^{x_k} = H_G$.

Consider the problem 17.41 from "Kourovka notebook" [1]:

*Let $H$ be a solvable subgroup of finite group $G$ and $G$ does not contain nontrivial normal solvable subgroups. Are there always exist five subgroups conjugated with $H$ such that their intersection is trivial?*

The problem is reduced to the case when $G$ is almost simple in [2]. Specifically, it is proved that if for each almost simple group $G$ and solvable subgroup $H$ of $G$ condition $Reg_H(G, 5) \geq 5$ holds then for each finite nonsolvable group $G$ and maximal solvable subgroup $H$ of $G$ condition $Reg_H(G, 5) \geq 5$ holds.

Let $p$ be a prime number and $q = p^t$. A cyclic irreducible subgroup $Sin_n(q)$ of $GL_n(q)$ of order $q^n - 1$ is called a *Singer cycle*. If $H$ is a cycle subgroup of $GU_n(q)$ and $|H| = q^n - (-1)^n$ we also call it a Singer cycle and denote by $Sin_n(q)$.

By $\varphi_n$ we denote an automorphism of $Sin_n(q)$ such that $\varphi_n : g \mapsto g^q$ if $G = GL_n(q)$ and $\varphi_i : g \mapsto g^{q^2}$ if $G = GU_n(q)$.

We have proved the following

**Theorem.** *Let $G$ be isomorphic to $GL_n(q)$ or $GU_n(q)$ and $H$ be a subgroup of $G$ such that $H$ is block diagonal with blocks isomorphic to $Sin_{n_i}(q) \rtimes \langle \varphi_{n_i} \rangle; i = 1, \ldots, k; \sum_{i=1}^{k} n_i = n$. Then $b_H(G) \leq 4$.*

### References

[1] Kourovka notebook, Edition 18, Novosibirsk, 2014.

[2] E. P. Vdovin, On the base size of a transitive group with solvable point stabilizer. *Journal of Algebra and Application* **11(1)** (2012) 14.