Completability of hamiltonian cycle in halved cube

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Halved cube $\frac{1}{2}Q_n$ – is a graph, whose vertex set is a set of all binary words of length n with even (odd) weight; two vertices are adjacent if Hamming distance between them equals 2. Halved cube graphs were studied in several papers $[1, 2]$. We consider hamiltonian cycles in halved cube graphs, in particularly, completability of these cycles. Let

$$C^{1/2} = v_1, v_2, v_3, \dots, v_{2^{n-1}-1}, v_{2^{n-1}}$$

be a hamiltonian cycle in $\frac{1}{2}Q_n$. We call this cycle *completable* if there are binary words $u_1, u_2, \dots u_{2^{n-1}}$ of length n such that cycle

$$C = v_1, \boldsymbol{u_1}, v_2, \boldsymbol{u_2}, v_3, \boldsymbol{u_3}, v_4, \ \dots \ , v_{2^{n-1}}, \boldsymbol{u_{2^{n-1}}}$$

is hamiltonian in Q_n.

For any $n \geq 4$ we prove existence of hamiltonian cycle in $\frac{1}{2}Q_n$, which is not completable.

For hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ we create auxiliary graph $\mathcal{G}\left(C^{1/2}\right)$. Without loss of generality we assume, that $C^{1/2}$ is a cycle, containing all binary words of length n with odd weight.

Vertex set of $\mathcal{G}(C^{1/2})$ – all binary words of length n with even weight. We call vertex $x \in V(\mathcal{G}(C^{1/2}))$ *closest vertex to edge* (u, v) of cycle $C^{1/2}$, if $d(u, v) = d(u, x) + d(x, v)$. Obviously, for any edge of $C^{1/2}$ there are exactly two closest vertices. Then, for every edge e of $C^{1/2}$, we add edge (v, u) to graph $\mathcal{G}$, where u and v are closest vertices to e.

Necessary and sufficient condition of cycle completability is stated in terms of auxiliary graph $\mathcal{G}\left(C^{1/2}\right)$:

Theorem. *Hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ is completable iff there is no trees among connected components of $\mathcal{G}(C^{1/2})$.*

References

[1] W. Imrich, S. Klavzar, A. Vesel, A characterization of halved cubes. *Ars Combin.* **48** (1998) 27–32.

[2] M. Deza, S. Shpectorov, Recognition of the l1-graphs with complexity [0(nm)], or football in a hypercube. *European J. Combin.* **17** (1991) 279–289.