Completability of hamiltonian cycle in halved cube

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Halved cube $\frac{1}{2}Q_n$ is a graph, whose vertex set is a set of all binary words of length $n$ with even (odd) weight; two vertices are adjacent if Hamming distance between them equals 2. Halved cube graphs were studied in several papers [1, 2]. We consider hamiltonian cycles in halved cube graphs, in particular, completability of these cycles. Let 

$$C^{1/2} = v_1, v_2, v_3, \ldots, v_{2n-1}, v_{2n-1}$$

be a hamiltonian cycle in $\frac{1}{2}Q_n$. We call this cycle completable if there are binary words $u_1, u_2, \ldots, u_{2n-1}$ of length $n$ such that cycle 

$$C = v_1, u_1, v_2, u_2, v_3, u_3, \ldots, v_{2n-1}, u_{2n-1}$$

is hamiltonian in $Q_n$.

For any $n \geq 4$ we prove existence of hamiltonian cycle in $\frac{1}{2}Q_n$, which is not completable.

For hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ we create auxiliary graph $G(C^{1/2})$. Without loss of generality we assume, that $C^{1/2}$ is a cycle, containing all binary words of length $n$ with odd weight.

Vertex set of $G(C^{1/2})$ – all binary words of length $n$ with even weight. We call vertex $x \in V(G(C^{1/2}))$ closest vertex to edge $(u, v)$ of cycle $C^{1/2}$, if $d(u, v) = d(u, x) + d(x, v)$. Obviously, for any edge of $C^{1/2}$ there are exactly two closest vertices. Then, for every edge $e$ of $C^{1/2}$, we add edge $(v, u)$ to graph $G$, where $u$ and $v$ are closest vertices to $e$.

Necessary and sufficient condition of cycle completability is stated in terms of auxiliary graph $G(C^{1/2})$:

**Theorem.** Hamiltonian cycle $C^{1/2}$ in $\frac{1}{2}Q_n$ is completable iff there is no trees among connected components of $G(C^{1/2})$.

References
