Groups with the minimal condition for non-abelian noncomplemented subgroups

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Remind that the group G is called Shunkov, if for any its finite subgroup K, every subgroup of the factor group $N_G(K)/K$, generated by two conjugate elements of prime order, is finite (V. D. Mazurov).

The class of periodic Shunkov groups is large and includes, for instance, the classes of 2-groups and binary finite groups. The class of locally graded groups is extremely large. The following new author's theorem holds.

Theorem. *Let G be a non-abelian periodic Shunkov group or a non-abelian locally graded group. Then G satisfies the minimal condition for non-abelian non-complemented subgroups iff it is a Chernikov group or an infinite periodic solvable group with complemented non-abelian subgroups.*

The known Olshanskiy's Examples of infinite simple groups with abelian proper subgroups (see, for instance, [1]) show that in this theorem the condition: "G is periodic Shunkov or locally graded" is essential. Note: the Shunkov groups with the minimal condition for abelian noncomplemented subgroups are completely described by N. S. Chernikov [2].

References

[1] A. Yu. Olshanskiy, Geometry of defining relations in groups, Nauka, Moskow, 1989. (in Russian).

[2] N. S. Chernikov, Shunkov groups with the minimal condition for noncomplemented abelian subgroups. *J. Sib. Federal Univ. Mathematics and Physics* **8(4)** (2015) 377–384.