Automorphisms of distance-regular graph with intersection array $\{121, 90, 1; 1, 30, 121\}$

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Suppose that Γ — antipodal distance-regular graph of diameter 3 with $\lambda = \mu$, in which the neighbourhood of each vertex is strongly regular with parameters (v', k', λ', μ'). In this case Γ has the intersection array $\{k, \mu(r-1), 1; 1, \mu, k\}$, and the spectrum $k^1, \sqrt{k}^f, -1^k, -\sqrt{k}^f$, where $k = v', \mu = (v'-k'-1)/(r-1)$ and $f = (k+1)(r-1)/2$. Further, the number $(v'+1)(r-1)$ is even. Makhnev A.A. and Samoilenko M.S. [1] selected parameters of strongly regular graphs with no more than 1000 vertices, satisfying these conditions. In this paper automorphisms of distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$ and of strongly regular graph with parameters $(121, 30, 11, 6)$ are investigated.

Theorem 1. *Let Γ be strongly regular graph with parameters $(121, 30, 11, 6)$, $G = \mathrm{Aut}(\Gamma)$, g element of prime order p of G and $\Omega = \mathrm{Fix}(g)$. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$ and one of the following holds:*

(1) *Ω is empty graph and $p = 11$;*

(2) *Ω is n-clique, either $n = 1$, $p = 3, 5$, or $n = 3t + 1$, $p = 3$ or $n = 2t + 1$ and $p = 2$;*

(3) *Ω is m-coclique, either $m = 3t + 1$, $p = 3$ or $m = 2t + 1$ and $p = 2$;*

(4) *Ω contains an edge and is the union of s isolated cliques, either $p = 3$, the number of vertices in maximal clique from Ω is congruent to 1 by module 3 and s is congruent to 1 by module 3, or $p = 2$, the number of vertices in maximal clique from Ω is odd and s is odd;*

(5) *if Ω contains $[a]$ for some vertex $a \in \Omega$, then $p \le 3$ and in the case $|\Omega| = 31$ we have $p = 3$;*

(6) *Ω contains geodesic 2-way, $p \le 7$ and in the case $p = 7$ subgraph Ω is strongly regular with parameters $(16, 9, 4, 6)$.*

Theorem 2. *Let Γ be a distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$, $G = \mathrm{Aut}(\Gamma)$, g be an element of prime order p of G and $\Omega = \mathrm{Fix}(g)$ contains s vertices in t antipodal classes. Then $\pi(G) \subseteq \{2, 3, 5, 7, 11, 13, 23, 61\}$ and one of the following holds:*

(1) *Ω is empty graph, $p = 2, 61$;*

(2) *Ω is the antipodal class of Γ, $p = 11$;*

(3) *Ω is a t-clique, $p = 3$ and $t = 2, 5, 8, 11$;*

(4) *$p = 23$, Ω is a distance-regular graph with intersection array $\{29, 21, 1; 1, 7, 29\}$, or $p = 13$ and Ω is a distance-regular graph with intersection array $\{17, 12, 1; 1, 4, 17\}$;*

(5) *$p = 7$, $t = 10, 17, 24$ and in the case $t = 10$ subgraph Ω is a distance-regular graph with intersection array $\{9, 6, 1; 1, 2, 9\}$;*

(6) *$p = 5$, $t = 2, 7, 12, 17, 22, 27$ and in the case $t = 7$ subgraph Ω is the union of four isolated 7-cliques;*

(7) *$p = 3, s = 4$, $t = 3l + 2$, $l \le 9$ and in the case $t = 5$ subgraph Ω is the union of four isolated 5-cliques;*

(8) *$p = 2, s > 0$, any vertex from $\Gamma - \Omega$ is adjacent with even number vertices in Ω and either $s = 2$, $t \le 60$, or $s = 4$, $t \le 30$.*

Corollary. *Let Γ be a vertex-symmetric distance-regular graph Γ with intersection array $\{121, 90, 1; 1, 30, 121\}$. Then Γ is the arc-transitive graph with the socle of automorphism group isomorphic to $Z_2 \times L_2(121)$.*

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References

[1] A. Makhnev, M. Samoilenko, On distance-regular covers of cliques with strongly regular neighbourhoods of vertices, Proceedings of the 46 International school-conference, Institute of Mathematics and Mechanics UB RAS, Yekaterinburg 2015, 13-18.