New infinite family of Cameron-Liebler line classes

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based on joint work with **Alexander Gavrilyuk**,
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G2S2, Novosibirsk, August 2016

Projective geometry

Let $V = GF(q)^{n+1}$.

- $PG(n,q)$ — n-dim. projective space over $GF(q)$

$$\sim : \mathbf{x} \sim \mathbf{y} \ (\mathbf{x}, \mathbf{y} \in V \setminus \{\mathbf{0}\}) \Leftrightarrow \exists \alpha \in GF(q) : \mathbf{x} = \alpha \mathbf{y}$$
$$PG(n,q) = (V \setminus \{\mathbf{0}\}) / \sim$$

- point of $PG(n,q)$
  1-dim. vector subspace of V
- line of $PG(n,q)$
  2-dim. vector subspace of V
- ...
- hyperplane
  n-dim. vector subspace of V
- spread — a line set partitioning the points of $PG(n,q)$

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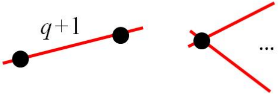
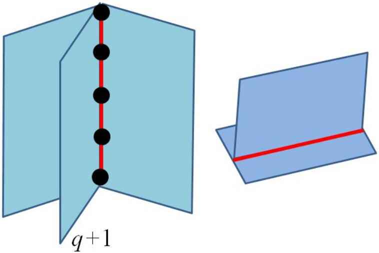
Projective geometry $PG(3, q)$

- $\exists!$ line through $\forall$ pair of points with exactly $q+1$ points on a line, while $\forall$ pair of lines has at most one point in common



- a line belongs to exactly $q+1$ planes, and $\exists!$ line in the intersection of $\forall$ pair of planes



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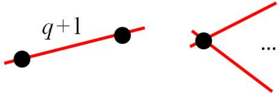

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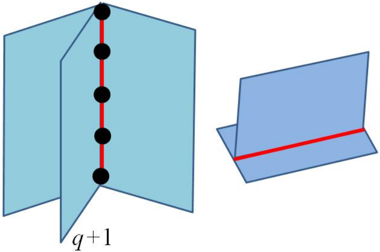

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Cameron-Liebler line class: definition

A **Cameron-Liebler line class** $\mathcal{L}$ is a set of lines of $PG(3,q)$ such that

$\exists$ a number x: for $\forall$ spread S

$$|\mathcal{L} \cap S| = x$$

The number x is called the **parameter** of $\mathcal{L}$.

One can show that $|\mathcal{L}| = x(q^2+q+1)$.

A line class $\overline{\mathcal{L}}$ complement to $\mathcal{L}$ is also a Cameron − Liebler line class with $x(\overline{\mathcal{L}}) = q^2+1-x(\mathcal{L}) \Rightarrow$ w.l.o.g. $x \leqslant \frac{q^2+1}{2}$.

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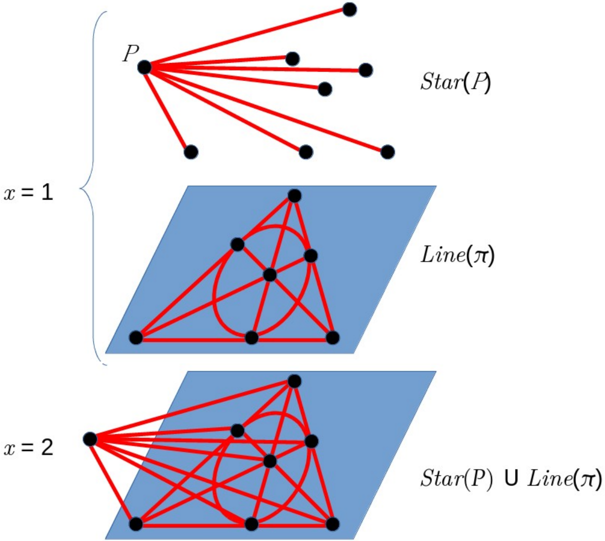
Cameron-Liebler line class: examples



Cameron-Liebler conjecture and motivation

Conjecture (Cameron, Liebler, 1982)

The only Cameron-Liebler line classes are those shown above (i.e., $x \notin \{3, \ldots, \frac{q^2+1}{2}\}$?).

- This problem is related to the problem of classification of the collineation groups of $PG(n, q)$ with the same number of orbits on points and lines.

  Solved by Bamberg and Penttila.

- The Cameron-Liebler line classes give rise to completely regular codes of strength 1 in the Grassmann graphs $J_q(4, 2)$.
- The Cameron-Liebler line classes give rise to some point sets in $PG(5, q)$ with two intersection numbers with respect to planes $\Rightarrow$ projective 2-weight codes $\Rightarrow$ strongly regular graphs.

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Counterexamples

- $x = 7$ in $PG(3,4)$.

(Govaerts, Penttila'05)

- In 2011 M. Rodgers constructed new Cameron – Liebler line classes for many odd values of q ($q < 200$) satisfying $q \equiv 1 \mod 4$ and $q \equiv 1 \mod 3$, having parameter $x = \frac{1}{2}(q^2 - 1)$.

  These new examples are made up of a union of orbits of a cyclic collineation group having order $q^2 + q + 1$.

Rodgers, 2011.

- $x = 10$ in $PG(3,5)$.

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- a new infinite family in $PG(3,q)$, $q \equiv 5$ or $9 \mod 4$, $x = (q^2 - 1)/2 \Rightarrow x = (q^2 + 1)/2$.

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Bound on x

It seems that the right question is about the lower bound for x.

- $x \neq 3, 4$ if $q \geqslant 5$.

  (Penttila'91)

- $x \notin \{3, \ldots, \sqrt{q}\}$.

  (Bruen, Drudge'98)

- $x \notin \{3, \ldots, e(q)\}$ where $q + 1 + e(q)$ is the size of the smallest non-trivial blocking set in $PG(2, q)$.

  (Drudge'99)

- $x \notin \{3, \ldots, q\}$.

  (Metsch'10)

- $x > cq^{4/3}$ (with some constant c).

  (Metsch'14)

- about a half of values from $\{3, \ldots, \frac{q^2+1}{2}\}$ cannot be the parameters of Cameron-Liebler line classes.

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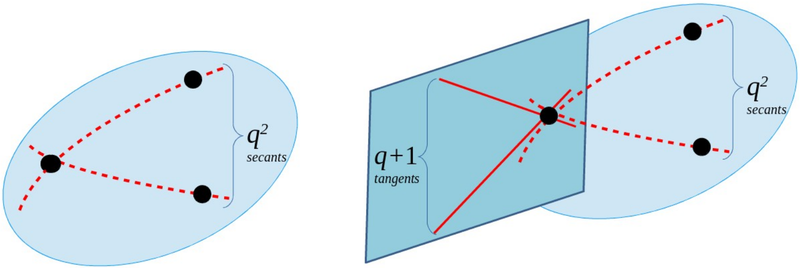
Elliptic quadric, q odd

The **elliptic quadric** Q is the set of zeros of the equation

$$Q(x_0, x_1, x_2, x_3) = -ax_0^2 + x_1^2 + x_2x_3 = 0,$$

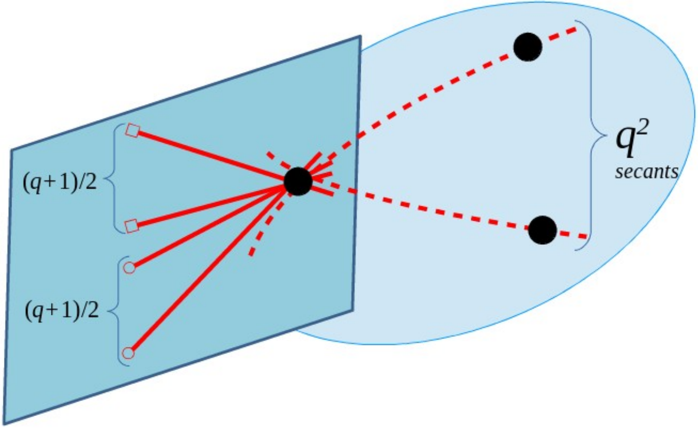
where a is any non-square in $GF(q)$.



- $q^2 + 1$ points;
- any line intersects Q in at most 2 points;
- there exists a unique tangent plane to any point of Q;

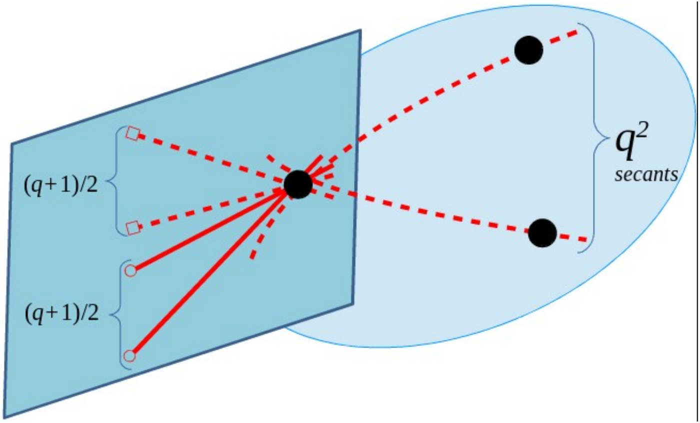
Bruen-Drudge family of C.-L. line classes

The set of $q+1$ tangents to a point of Q can be partitioned into two groups, depending on whether there exists a point P on a tangent with $Q(P)$ a square or non-square in $GF(q)$.



Bruen-Drudge family of C.-L. line classes

A Cameron-Liebler line class of the Bruen-Drudge family consists of all secants to Q, and a half of tangents that all correspond to squares (or non-squares).



Switching

Lemma (Penttila)

Let $\mathcal{L}$ be a Cameron-Liebler line class such that there exists an incident point-plane pair (P, π) satisfying the following conditions:

- $\mathsf{Line}(\pi) \setminus \mathsf{Star}(P) \not\subseteq \mathcal{L}$,
- $\mathsf{Star}(P) \setminus \mathsf{Line}(\pi) \subseteq \mathcal{L}$.

Then

$$\mathcal{L} \cup (\mathsf{Line}(\pi) \setminus \mathsf{Star}(P)) \setminus (\mathsf{Star}(P) \setminus \mathsf{Line}(\pi))$$

is a Cameron-Liebler line class $\mathcal{L}'$ with the same parameter.

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Switching: proof

$$\mathcal{L}' := \mathcal{L} \cup (\mathsf{Line}(\pi) \setminus \mathsf{Star}(P)) \setminus (\mathsf{Star}(P) \setminus \mathsf{Line}(\pi))$$

Proof:

For any spread S:
either

$$S \text{ contains a line } \underline{\text{on } P \text{ and in } \pi} \Rightarrow S \cap \mathcal{L}' = S \cap \mathcal{L},$$

or

$$S \text{ contains a line } \underline{\ell \in \pi,\ P \notin \ell}, \text{ and a line } \underline{m \ni P,\ m \notin \pi} \Rightarrow S \cap \mathcal{L}' = (S \cap \mathcal{L}) \cup \{m\} \setminus \{\ell\}.$$

Thus,

$$|S \cap \mathcal{L}'| = |S \cap \mathcal{L}| = x.$$

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Switching: application

There exists an incident point-plane pair (P, π) satisfying the following conditions:

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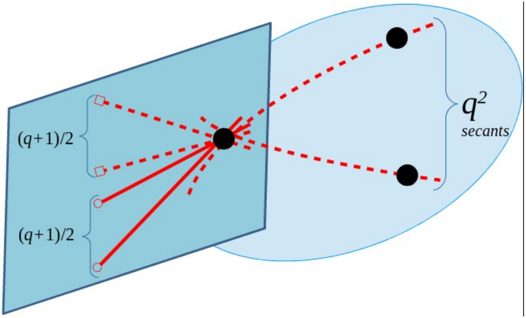

Switching with respect to a point of Q and its tangent plane produces a new Cameron-Liebler line class with $x = \frac{q^2+1}{2}$.

Switching $\Rightarrow x = \frac{q^2+1}{2}$

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Then the parameter x of $\mathcal{L}$ is equal to $\frac{q^2+1}{2}$.

In $PG(3,5)$ with $x = \frac{q^2+1}{2}$:

- the Bruen-Drudge example;
  $\Updownarrow$
- the switched Bruen-Drudge example;
- the Rodgers example;

Switching $\Rightarrow x = \frac{q^2+1}{2}$

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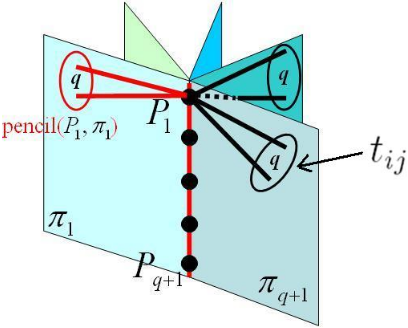
Patterns (G. & Mogilnykh, 2012)

Let l be a line of $PG(3,q)$, $\mathcal{L}$ a Cameron – Liebler line class. Consider all the points P_i, $i = 1, \ldots, q+1$ that are on l, and all the planes π_j, $j = 1, \ldots, q+1$ that contain l.



Define a square matrix T of order $q+1$ whose (i,j)-element is $|\text{pencil}(P_i, \pi_j) \cap \mathcal{L} \setminus \{l\}|$

We will call such matrix a pattern w.r.t. l.

Properties of patterns

Let $T := (t_{ij})$ be a pattern w.r.t. a line l, and define

$$\chi := \begin{cases} 0 \text{ if } l \notin \mathcal{L}, \\ 1 \text{ if } l \in \mathcal{L}, \end{cases}$$

Then the following hold:

- $t_{ij} \in \mathbb{N}$, $0 \leq t_{ij} \leq q$ for all $i, j \in \{1, \ldots, q+1\}$;
- $\sum_{i,j=1}^{q+1} t_{ij} = x(q+1) + \chi(q^2 - 1)$;
- $\sum_{j=1}^{q+1} t_{kj} + \sum_{i=1}^{q+1} t_{il} = x + (q+1)(t_{kl} + \chi), \forall k, l$;
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Properties of patterns

Let $T := (t_{ij})$ be a pattern w.r.t. a line l, and define

$$\chi := \begin{cases} 0 \text{ if } l \notin \mathcal{L}, \\ 1 \text{ if } l \in \mathcal{L}, \end{cases}$$

Then the following hold:

- $t_{ij} \in \mathbb{N}$, $0 \leq t_{ij} \leq q$ for all $i, j \in \{1, \ldots, q+1\}$;
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Some history and acknowledgements

- This family of Cameron-Liebler line classes together with the switching was found by Tim Penttila about 10 years ago, but it has not been published.
- With Ilia Matkin, we re-discovered this family during his visit at USTC (May of 2016).
- After that I realised that in 2015 during the CoCoA (Combinatorics and Comp. Algebra) conference organised by Anton Betten, Tim told me that there was a C.-L. line class arising from the Bruen-Drudge example by something like switching operation.
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