On the spectra of non-commuting graphs

Modjtaba Ghorbani  
Department of Mathematics, Shahid Rajaee Teacher Training University, Tehran, I. R. Iran  
mghorbani@srttu.edu

Maryam Jalali-Rad  
Department of Pure Mathematics, Faculty of Mathematical Sciences, University of Kashan, Kashan, I. R. Iran  
jalali6834@srttu.edu

All graphs considered in this paper are simple and finite. Also, all groups are finite and non-abelian. There are a number of constructions of graphs from groups or semi-groups in the literature. Let $G$ be a non-abelian group with center $Z(G)$. The non-commuting graph (NC-graph) $\Gamma(G)$ is a simple and undirected graph with the vertex set $G \setminus Z(G)$ and two vertices $x, y \in G \setminus Z(G)$ are adjacent whenever $xy \neq yx$. The concept of NC-graphs was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975. For background materials about non-commuting graphs, we encourage the reader to see reference [1]. The non-commuting graph $\Gamma(G)$ of group $G$ was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975, see [2]. In this article, we prove that regular non-commuting graphs are Eulerian. We also prove that there is no $2^s q$-regular non-commuting graph, where $q$ is a prime number greater than 2.

An integral graph is a graph with integral spectrum. 

\textbf{Theorem.} If $\Gamma(G)$ is $k$-regular integral non-commuting graph where $k \leq 16$, then $k = 4$ and $G \cong D_8, Q_8$ or $k = 8$ and $G \cong \mathbb{Z}_2 \times D_8, \mathbb{Z}_2 \times Q_8, SU(2), M_{16}, \mathbb{Z}_4 \times \mathbb{Z}_4, \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ or $k = 16$ and $G \cong \text{SmallGroup}(32,i)$, where 

\[ i \in \{2, 4, 5, 12, 17, 22, 23, 24, 25, 26, 37, 38, 46, 47, 48, 49, 50\}. \]

\textbf{References}
