On the spectra of non-commuting graphs

Modjtaba Ghorbani
Department of Mathematics, Shahid Rajaee Teacher Training University, Tehran, I. R. Iran
mghorbani@srttu.edu

Maryam Jalali-Rad
Department of Pure Mathematics, Faculty of Mathematical Sciences,
University of Kashan, Kashan, I. R. Iran
jalali6834@srttu.edu

All graphs considered in this paper are simple and finite. Also, all groups are finite and non-abelian. There are a number of constructions of graphs from groups or semi-groups in the literature. Let G be a non-abelian group with center $Z(G)$. The non-commuting graph (NC-graph) $\Gamma(G)$ is a simple and undirected graph with the vertex set $G \setminus Z(G)$ and two vertices $x, y \in G \setminus Z(G)$ are adjacent whenever $xy \neq yx$. The concept of NC-graphs was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975. For background materials about non-commuting graphs, we encourage the reader to see reference [1]. The non-commuting graph $\Gamma(G)$ of group G was first considered by Paul Erdős to answer a question on the size of the cliques of a graph in 1975, see [2]. In this article, we prove that regular non-commuting graphs are Eulerian. We also prove that there is no $2^s q-$regular non-commuting graph, where q is a prime number greater than 2.

An integral graph is a graph with integral spectrum.

Theorem. *If $\Gamma(G)$ is $k-$regular integral non-commuting graph where $k \leq 16$, then $k = 4$ and $G \cong D_8, Q_8$ or $k = 8$ and $G \cong \mathbb{Z}_2 \times D_8, \mathbb{Z}_2 \times Q_8, SU(2), M_{16}, \mathbb{Z}_4 \ltimes \mathbb{Z}_4, \mathbb{Z}_4 \ltimes \mathbb{Z}_2 \times \mathbb{Z}_2$ or $k = 16$ and $G \cong SmallGroup(32, i)$, where*

$$i \in \{2, 4, 5, 12, 17, 22, 23, 24, 25, 26, 37, 38, 46, 47, 48, 49, 50\}.$$

References

[1] A. Abdollahi, S. Akbari, H. R. Maimani, Non-commuting graph of a group. *J. Algebra* **298** (2006) 468–492.

[2] B. H. Neumann, A problem of Paul Erdős on groups. *J. Austral. Math. Soc.* **21A** (1976) 467–472.