Splitting planar graphs of bounded girth to subgraphs with short paths

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A graph $G = (V, E)$ is (a, b)-*partitionable* for positive integers a, b if its vertex set can be partitioned to subsets V_1 and V_2 such that induced subgraphs $G[V_1]$ and $G[V_2]$ do not contain paths of length exceeding $a-1$ and $b-1$ respectively. Mihok [4] showed that for any constants a and b their exists series of planar graphs which are not (a, b)-partitionable. However, all examples of graphs constructed by Mihok contain many 3-cycles. On the other hand, for planar graphs with sufficiently large girth it was established in series of papers that they are (a, b)-partitionable for small a and b. For example, in [2] it was proved that any planar graph with girth at least 7 is (2,2)-partitionable. Therefore, a question arises: what is the smallest integer g such that there exist positive integers a, b with the property that any planar graph with girth at least g is (a, b)-partitionable?

For planar graphs with girth at least 6, it was recently proved that they are (5,5)-partitionable [3] and that the vertex set of any such a graph can be partitioned to subsets V_1 and V_2 such that both subgraphs $G[V_1]$ and $G[V_2]$ are linear forests whose paths have length at most 14 [1]. Another important result in [1] is a construction of series of planar graphs with girth 4 which are not (a, b)-partitionable for any given a and b. So it follows by the results in [1, 3], that $5 \le g \le 6$.

In this paper we make the final step in determining g by proving that any planar graph with girth at least 5 is (7,7)-partitionable. Hence we establish that $g = 5$. Furthermore, we prove the list version of our main result: if every vertex v of a graph is given a list $L(v)$ of two colours then we can colour the graph vertices from their list in such a way that each monochromatic component is a tree of diameter at most 6.

The work is supported by RFBR (projects 15-01-00976 and 15-01-05867).

References

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