

On Deza graphs with disconnected second neighbourhoods of vertices*Sergey Goryainov**Chelyabinsk State University, Chelyabinsk, Russia**N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia*
44g@mail.ru*Galina Isakova**N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia*
carleink@gmail.com*Vladislav Kabanov**N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia*
Ural Federal University, Yekaterinburg, Russia
vvk@imm.uran.ru*Natalia Maslova**N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia*
Ural Federal University, Yekaterinburg, Russia
butterson@mail.ru*Leonid Shalaginov**Chelyabinsk State University, Chelyabinsk, Russia*
leonidshalaginov@rambler.ru

We consider finite undirected graphs without loops and multiple edges.

A graph Γ is called a *Deza graph* if it is regular and the number of common neighbours of two distinct vertices takes on one of two values. A Deza graph Γ is called *strictly Deza graph* if it has diameter 2 and is not strongly regular. Let x be a vertex of a Deza graph Γ . The subgraph of the graph Γ induced by vertices which are at distance 2 from the vertex x is called *second neighbourhood* of the vertex x .

In 1992, Gardiner, Godsil, Hensel, Royle [1] proved that a strongly regular graph which contains a vertex with disconnected second neighbourhood is a complete multipartite graph with parts of the same size greater or equal to 2.

Alexander Gavriluyk proposed to study strictly Deza graphs which contain a vertex with disconnected second neighbourhood.

Let Γ be a vertex transitive strictly Deza graph such that the second neighbourhood of each its vertex is disconnected. It was proved [2, Theorem 1] that Γ is either edge regular or coedge regular. Also, there were obtained a characterization of a strictly Deza graph which contain a vertex with disconnected second neighbourhood in cases of edge regularity and coedge regularity of this graph.

In this work we proved the following theorem which is a generalization of the previous result [2, Theorem 1].

Theorem. *Let Γ be a strictly Deza graph such that the second neighbourhood of each its vertex is disconnected. Then Γ is either edge regular or coedge regular.*

Acknowledgment. The work is supported by RFBR (grant no. 16-31-00316). The third author is supported by the grant of the President of Russian Federation for young scientists (grant no. MK-6118.2016.1) and is a winner of the competition for young mathematicians of the Dmitry Zimin Foundation YDynastyY in 2013 year.

References

- [1] A. D. Gardiner, C. D. Godsil, A. D. Hensel, G. F. Royle, Second neighbourhoods of strongly regular graphs. *Discrete Mathematics* **103** (1992) 161–170.
- [2] S. V. Goryainov, G. S. Isakova, N. V. Maslova, L. V. Shalaginov, On the Deza graphs with disconnected second neighbourhood. *Abstracts of the Russian-Slovenian Workshop "Groups and Graphs, Cycles and Coverings"* Novosibirsk: Sobolev Institute of Mathematics, September 23–26 (2014) 9.