New construction of Deza graphs

Sergey Goryainov
Chelyabinsk State University, Chelyabinsk, Russia
N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia
44g@mail.ru

This is joint work with Alexander Gavrilyuk, Galina Isakova and Leonid Shalaginov

A graph is called *regular* of valency k, if each of its vertices has exactly k neighbours. A graph is called a *Deza* graph with parameters (n, k, b, a), $b \geq a$, if it has n vertices, is regular of valency k, and the number of common neighbours of any two of its vertices belongs to the set $\{a, b\}$. A Deza graph is called a *strictly Deza* graph, if it has diameter 2 and is not strongly regular.

Let G be a finite group. Let S be a non-empty subset of G such that $1_G \notin S$ and, for any $s \in S$, one also has that $s^{-1} \in S$. A graph $Cay(G, S)$ with the vertex set G and the adjacency defined by $x \sim y \Leftrightarrow xy^{-1} \in S, \ \forall x, y \in G$, is called a Cayley graph of the group G with generating set S.

Recall that the Paley graph of order q, where $q \equiv 1 (\mathrm{mod}\ 4)$, is a Cayley graph $Cay(\mathbb{F}_q^+, S_q)$, which is strongly regular with parameters $(n, k, \lambda, \mu) = (q, \frac{q-1}{2}, \frac{q-5}{4}, \frac{q-1}{4})$. Here $\mathbb{F}_q^+$ and S_q are the additive group and the set of non-zero squares of the finite field $\mathbb{F}_q$ of order q, respectively.

Let q_1, q_2 be two odd prime powers such that $q_2 - q_1 = 4$. Let $\overline{S}_{q_1} := \mathbb{F}_{q_1}^* \setminus S_{q_1}$ and $\overline{S}_{q_2} := \mathbb{F}_{q_2}^* \setminus S_{q_2}$ be the sets of non-squares in the corresponding fields. Let $S_0 := \{(0, x) \mid x \in \mathbb{F}_{q_2}^*\}$, $S_1 := S_{q_1} \times \overline{S}_{q_2}$ and $S_2 := \overline{S}_{q_1} \times S_{q_2}$. In this talk we will discuss the following theorem and some related results.

Theorem. *The graph* $Cay(\mathbb{F}_{q_1}^+ \times \mathbb{F}_{q_2}^+, S_0 \cup S_1 \cup S_2)$ *is a strictly Deza graph with parameters* $(v, \frac{v+3}{2}, \frac{v+7}{4}, \frac{v+3}{4})$, *where* $v = q_1 q_2$.

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