On vertex connectivity of Deza graphs

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We consider finite undirected graphs without loops and multiple edges. A graph is called *regular* of valency k, if each of its vertices has exactly k neighbours. A graph is called a *Deza* graph with parameters (n, k, b, a), $b \geq a$, if it has n vertices, is regular of valency k, and the number of common neighbours of any two of its vertices belongs to the set $\{a, b\}$. A Deza graph is called a *strictly Deza* graph, if it has diameter 2 and is not strongly regular. *The vertex connectivity* $\kappa(\Gamma)$ of a connected graph Γ is the minimum number of vertices one has to remove in order to make the graph Γ disconnected (or empty).

In 1985, Brouwer and Mesner proved [1] that the vertex connectivity of a strongly regular graph is equal to its valency. In 2009, Brouwer and Koolen generalized [2] this result to the class of distance-regular graphs.

Vertex connectivity of strictly Deza graphs obtained from the construction [3, theorem 3.1] based on strongly regular graphs was studied in [4]. The case of the eigenvalue $r \leq 2$ remained open.

In this work we study the vertex connectivity of strictly Deza graphs obtained from strongly regular graphs with eigenvalue $r = 1$ (i.e. from complements to Seidel strongly regular graphs). Note that the construction [3, theorem 3.1] requires the existence of involutive automorphism of a strongly regular graph which interchanges the only non-adjacent vertices. Such automorphisms in the case of the complements to the triangular and lattice graphs have been studied in [5].

In this work the following results were obtained:

Theorem 1. *Let Δ be a strictly Deza graph obtained from either the complement to the triangular graph $T(n)$, $n \geq 3$, or from the complement to one of the following sporadic graphs: Petersen graph, Shrikhande graph, Clebsch graph, Schlafli graph, Chang graph. Then the vertex connectivity of Δ is equal to k, where k is the valency of the graph Δ.*

Theorem 2. *Let Δ be a Deza graph obtained from the complement to $n \times n$-lattice. Then the vertex connectivity of Δ is equal to $k - 1$, where k is the valency of the graph Δ.*

Note that in the case of the complement to $n \times n$-lattice, where n is even, and the automorphism which interchanges $n/2$ pairs of rows the construction [3, theorem 3.1] gives vertex-transitive (moreover, Cayley) strictly Deza graphs.

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References

[1] A. E. Brouwer, D. M. Mesner, The connectivity of strongly regular graphs. *Europ. J. Combin* **6** (1985) 215–216.

[2] A. E. Brouwer, J. H. Koolen, The vertex-connectivity of a distance-regular graph. *Europ. J. Combin* **30(3)** (2009) 668–673.

[3] M. Erickson, S. Fernando, W. H. Haemers, D. Hardy, J. Hemmeter, Deza graphs: A generalization of strongly regular graphs. *J. Comb. Des* **7(6)** (1999) 359–405.

[4] A. L. Gavrilyuk , S. V. Goryainov, V. V. Kabanov, On the vertex connectivity of Deza graphs. Proceedings of the Steklov Institute of Mathematics, June 2014, 285, Supplement 1, 68–77.

[5] S. V. Goryainov, L. V. Shalaginov, On Deza graphs with parameters of complement graphs to lattice and triangular graphs. *Diskretn. Anal. Issled. Oper.* **20(2)** (2013) 3–14 (in Russian).