A construction of infinite families of directed strongly regular graphs

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The main subject of this talk is directed strongly regular graph, a possible generalization of the well-known (undirected) strongly regular graphs for the directed case, introduced by Duval in [1]. A *directed strongly regular graph* (DSRG) with parameters (n, k, t, λ, μ) is a regular directed graph on n vertices with valency k such that every vertex is incident with t undirected edges; the number of directed paths of length 2 directed from a vertex x to another vertex y is λ if there is an arc from x to y; and μ otherwise.

Using structural analysis of DSRGs we observed that a significant amount of them can be obtained from smaller DSRGs with the aid of suitably defined graph product (π-join) which is based on a partition of the vertex set of the smaller graph. Inspired by this observation we derived the necessary and sufficient conditions to solve, when a homogeneous partition π of a DSRG altogether with π-join construction lead to a bigger DSRG. In fact, the partition π has to be equitable with a prescribed quotient matrix depending just on the parameters of the small DSRG. Using this approach we constructed dozens of infinite families of DSRGs. According to the catalogue of parameter sets with order at most 110, located on the webpage of A. Brouwer and S. Hobart (see [2]), we confirm the existence of DSRGs for 29 open parameter sets.

References

[1] A. M. Duval, A directed graph version of strongly regular graphs. *J. Combin. Th. A* **47** (1988) 71–100.

[2] A. E. Brouwer, S. Hobart, Tables of directed strongly regular graphs. `http://homepages.cwi.nl/~aeb/`.