On Deza graphs with parameters (v, k, k-1, a)

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By "graph" we mean "an undirected graph without loops and multiple edges". A graph Γ on v vertices is strongly regular with parameters (v, k, λ, μ) if it is regular of degree k, the number of common neighbors of two adjacent vertices is equal to λ and the number of common neighbors of two non-adjacent vertices is equal to μ (see, for example, [1]). A graph Γ on v vertices is a Deza graph with parameters (v, k, b, a), where $v > k \geq b \geq a \geq 0$, if it is regular of degree k and the number of common neighbors of two distinct vertices takes on one of two values a or b, not necessarily depending on the adjacency of the two vertices (see [2]). A strictly Deza graph is a Deza graph which is not strongly regular and has diameter 2.

Let Γ be a strongly regular graph with parameters (v, k, λ, μ). It's not difficult to see, that
(1) if $\mu = k$ then Γ is a complete multipartite graph;
(2) if $\mu = k - 1$ then Γ is the pentagon;
(3) if $\lambda = k - 1$ then Γ is an union of cliques.

Let Γ_1 and Γ_2 be graphs. Γ_2-extension of Γ_1 is a graph obtained by replacing vertices of Γ_1 by copies of Γ_2 and joining all edges between vertices from distinct copies of Γ_2 whenever the correspondent vertices of Γ_1 were adjacent.

In [2] it was obtained a result analogue to (1) for Deza graphs. It was proved, Γ is a strictly Deza graph with parameters (n, k, k, a) if and only if Γ is isomorphic to n_2-coclique extension of a strongly regular graph Γ_1 with parameters (n_1, k_1, λ, μ) for some n_1, k_1, λ, μ and n_2, where $\lambda = \mu$ and $n_2 \geq 2$. Our aim is to obtain results analogue to (2) and (3) for Deza graphs. We prove the following theorem.

Theorem. *A graph Γ is a strictly Deza graph with parameters $(v, k, k-1, a)$ if and only if Γ is isomorphic to 2-clique extension either of a complete multipartite graph or of a strongly regular graph with parameters $(\frac{v}{2}, \frac{k-1}{2}, \frac{a-2}{2}, \frac{a}{2})$.*

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References

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