On Deza graphs with parameters \((v, k, k-1, a)\)

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By “graph” we mean “an undirected graph without loops and multiple edges”. A graph \(\Gamma\) on \(v\) vertices is strongly regular with parameters \((v, k, \lambda, \mu)\) if it is regular of degree \(k\), the number of common neighbors of two adjacent vertices is equal to \(\lambda\), and the number of common neighbors of two non-adjacent vertices is equal to \(\mu\) (see, for example, [1]). A graph \(\Gamma\) on \(v\) vertices is a Deza graph with parameters \((v, k, b, a)\), where \(v > k \geq b \geq a \geq 0\), if it is regular of degree \(k\) and the number of common neighbors of two distinct vertices takes on one of two values \(a\) or \(b\), not necessarily depending on the adjacency of the two vertices (see [2]). A strictly Deza graph is a Deza graph which is not strongly regular and has diameter 2.

Let \(\Gamma\) be a strongly regular graph with parameters \((v, k, \lambda, \mu)\). It’s not difficult to see, that

1. if \(\mu = k\) then \(\Gamma\) is a complete multipartite graph;
2. if \(\mu = k - 1\) then \(\Gamma\) is the pentagon;
3. if \(\lambda = k - 1\) then \(\Gamma\) is an union of cliques.

Let \(\Gamma_1\) and \(\Gamma_2\) be graphs. \(\Gamma_2\)-extension of \(\Gamma_1\) is a graph obtained by replacing vertices of \(\Gamma_1\) by copies of \(\Gamma_2\) and joining all edges between vertices from distinct copies of \(\Gamma_2\) whenever the correspondent vertices of \(\Gamma_1\) were adjacent.

In [2] it was obtained a result analogue to (1) for Deza graphs. It was proved, \(\Gamma\) is a strictly Deza graph with parameters \((n, k, k, a)\) if and only if \(\Gamma\) is isomorphic to \(n_2\)-clique extension of a strongly regular graph \(\Gamma_1\) with parameters \((n_1, k_1, \lambda, \mu)\) for some \(n_1, k_1, \lambda, \mu\) and \(n_2\), where \(\lambda = \mu\) and \(n_2 \geq 2\).

Our aim is to obtain results analogue to (2) and (3) for Deza graphs. We prove the following theorem.

**Theorem.** A graph \(\Gamma\) is a strictly Deza graph with parameters \((v, k, k-1, a)\) if and only if \(\Gamma\) is isomorphic to 2-clique extension either of a complete multipartite graph or of a strongly regular graph with parameters \((\frac{v}{2}, \frac{k-1}{2}, \frac{a-2}{2}, \frac{a}{2})\).

**Acknowledgment.** The work is supported by RFBR (grant no. 16-31-00316). The second author is supported by the grant of the President of Russian Federation for young scientists (grant no. MK-6118.2016.1) and is a winner of the competition for young mathematicians of the Dmitry Zimin Foundation “Dynasty” in 2013 year.

**References**
