# On claw-free strictly Deza graphs

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# Definitions

**Definition.** Let $v,\ k,\ b$ and $a$ be integers such that $0 \leq a \leq b \leq k < v$. A graph $G$ is a *Deza graph* with parameters $(v, k, b, a)$ if

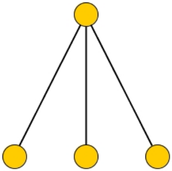
- $G$ has exactly $v$ vertices;
- for any vertex $u$ in $G$ its neighbourhood $N(u)$ has exactly $k$ vertices;
- for any two distinct vertices $u, w$ in $G$ the intersection $N(u) \cap N(w)$ takes on one of two values $b$ and $a$.

**Definition.** A *strictly Deza graph* is a Deza graph of diameter 2 that is not strongly regular.

**Definition.** The *inflation* of a graph $G$ is line graph of such graph which obtained from G by replacing each edge by path of length 2.

# Definitions

**Definition.** A star $S_k$ is the complete bipartite graph $K_{1,k}$. A star with $k = 3$ is called a *claw*.



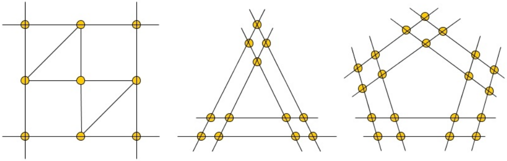
**Definition.** A *claw-free graph* is a graph that does not have a claw as an induced subgraph.

**Definition.** The *line graph* of a graph $G$ is another graph $L(G)$ that represents the adjacencies between edges of $G$.

## Previous results

**Theorem 1.** A graph $G$ is strictly Deza line graph if and only if it is

- the $4 \times n$ lattice graph, where $n > 1$ and $n \neq 4$ or
- one of the graphs $M_1$, $M_2$ or $M_3$.



$M_1$ - (9,4,2,1) $M_2$ - (12,6,3,2) $M_3$ - (20,6,2,1)

**Theorem 2.** Let $G$ be a connected claw-free Deza graph with diameter greater than 2. Then $G$ is one of that graphs:

- the inflation of cubic graph;
- line graph of triangle-free cubic graph;
- n-gon, where $n \geq 6$;
- the icosahedron.

# Previous results and Main aim

**Theorem 3.** Let $G$ be a strictly Deza graph, and there are following conditions:

1. $G$ is a claw-free graph;
2. $G$ contain 3-coclique;
3. $G$ is a union of closed neighborhoods of two nonadjacent vertices;

then $G$ is Deza graph with parameters $(9, 4, 2, 1)$ or $(12, 6, 3, 2)$.

Main aim - classification of claw-free Deza graphs with 3-coclique

# Consolidated result

Maria Chudnovsky and Paul Seymour "The structure of claw-free graphs"(November 2005)

**Theorem.** Let $G$ be claw-free and connected. Then either

- $G \in S_0 \cup ... \cup S_6$, or
- $G$ admit either a homogeneous pair of cliques, a $1$-join, a generalized $2$-join, or a hex-join, or
- $G$ is antiprismatic.

Conclusion:

- theorem is not convenient to use, especially for graphs with regularity conditions
- there are description (not construction) for some classes

# Used articles

[1] V.V. Kabanov, Siberian Mathematical Journal, 1998, Volume 39, Issue 5, pp 908–912

Description of connected claw-free graphs which contains a 3-coclique and every $\mu$-graph has radius greater than 1.

[2] V.V. Kabanov, A.A. Makhnev, Mathematical Notes, 1996, Volume 60, Issue 4, pp 372–377

Description of claw-free coedge regular graphs.

[3] A.A. Makhnev, Mathematical Notes, 1988, Volume 63, Issue 3, pp 357–362

Description of connected reduced claw-free graphs containing a 3-coclique, and all of whose $\mu$-subgraphs are regular of valency $\alpha > 0$.

# Main result

**Theorem.** *Let $G$ be a $K_{1,3}$-free strictly Deza graphs, and any two of its non-adjacent vertices belong to $3$-coclique. Then $G$ is one of that graphs:*

1. *the $4 \times n$-lattice, where $n > 2$, $n \neq 4$;*
2. *the $2$-extension of $3 \times 3$-lattice;*
3. *line $(20, 6, 2, 1)$-Deza graph.*

# Proof

**Lemma 1.** Let graph $G$ satisfies all conditions of the theorem and every $\mu$-subgraph has radius greater than $1$. Than $G$ is one of following graphs:

- the $4 \times n$-lattice, where $n > 2$, $n \neq 4$;
- the $2$-extension of $3 \times 3$-lattice.

**Lemma 2.** Let graph $G$ satisfies all conditions of the theorem and is a coedge regular graphs. Then $G$ is one of the cases from the conclusion of the lemma 1.

Further, we assume that there is a $\mu$-subgraph with radius $1$ and there are $\mu$-subgraphs both size $a$ and $b$.

# Proof

Let $\gamma$ and $\delta$ are vertexes of $G$ such that $\gamma \nsim \delta$ and $\mu$-subgraphs $\gamma \cap \delta$ has radius $1$.
Let vertex $\varepsilon \in \gamma \cap \delta$ be adjacent with all vertexes in $\mu$-subgraphs.

Introduce notations:

- $y = |[\gamma] \cap [\varepsilon]|$;
- $z = |[\delta] \cap [\varepsilon]|$;
- $x = |[\gamma] \cap [\delta] \cap [\varepsilon]|$.

# Proof. Possible cases

If $|\gamma \cap \delta| = a$, than $x = a - 1$

- If $y = z = a$ than $k = a + 3$
- If $y = a, z = b$ than $k = b + 3$
- If $y = z = b$ than $k = 2b - a + 3$

If $|\gamma \cap \delta| = b$, than $x = b - 1$

- If $y = z = a$ than $k = 2a - b + 3$
- If $y = a, z = b$ than $k = a + 3$
- If $y = z = b$ than $k = b + 3$

## Proof. Possible cases

**Lemma 3.** Let $k = a + 3$ than there is no appropriate parameters $(v, k, b, a)$ for $G$.

**Lemma 4.** Let $k = 2a - b + 3$ than there is no appropriate parameters $(v, k, b, a)$ for $G$.

**Lemma 5.** Let $k = b + 3$ than there is no appropriate parameters $(v, k, b, a)$ for $G$.

**Lemma 6.** Let $k = 2b - a + 3$ than $G$ is $(20, 6, 2, 1)$-Deza graph.

Thank you for attention!