On claw-free strictly Deza graphs

V. V. Kabanov
N.N. Krasovskii Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia
vvk@imm.uran.ru

A. V. Mityanina
Chelyabinsk State University, Chelyabinsk, Russia
nastya.mityanina@gmail.com

All graphs under consideration are undirected graphs without loops and multiple edges.

Let v, k, b and a be integers such that $0 \leq a \leq b \leq k < v$. A graph G is a *Deza graph* with parameters (v, k, b, a) if

- G has exactly v vertices;
- for any vertex u in G its neighbourhood $N(u)$ has exactly k vertices;
- for any two distinct vertices u, w in G the intersection $N(u) \cap N(w)$ takes on one of two values b and a.

The key of difference between a strongly regular graph and a Deza graph is that the size of $N(u) \cap N(w)$ does not necessary depend on adjacency u and w.

A *strictly Deza graph* is a Deza graph of diameter 2 that is not strongly regular.

The study of strongly regular graphs has a long history (see for example [1]), and the study of strictly Deza graphs started since the paper written by five authors M. Erickson, S. Fernando, W. H. Haemers, D. Hardy and J. Hemmeter [2] had been published.

A subset of the vertices of a graph is called *coclique* if there does not exist adjacent vertices. The *complete bipartite graph* $K_{m,n}$ is a graph which set of vertices can be divided into subsets of the cardinalities m and n such that each vertex in one subset is adjacent to every vertex in the other subset and to no vertex in its own set. A claw is another name for the complete bipartite graph $K_{1,3}$. A *claw-free graph* is a graph that does not have a claw as an induced subgraph. Claw-free graphs were initially studied as a generalization of line graphs. The *line graph* of a graph G is another graph $L(G)$ that represents the adjacencies between edges of G.

In [3] it was described the class of strictly Deza line graphs. In [4] it was described the class of claw-free strictly Deza graphs which are the union of closed neighborhoods of some two non-adjacent vertices, that is there are some two distinct vertices aren't belonging to any 3-coclique.

In this work we proved the following theorem.

Theorem. *Let G be a claw-free strictly Deza graph, and any two of its non-adjacent vertices belong to 3-coclique. Then G is one of that graphs:*

1. *the* $4 \times n$*-lattice, where* $n > 2$, $n \neq 4$;
2. *the* 2*-extension of* 3×3*-lattice;*
3. *Deza line graph with parameters* $(20, 6, 3, 2)$.

References

[1] A. E. Brouwer, A. M. Cohen, A. Neumaier, *Distance-Regular Graphs*, Springer–Verlag, New York, 1989.

[2] M. Erickson, S. Fernando, W. H. Haemers, D. Hardy, J. Hemmeter, Deza graphs: a generalization of strongly regular graphs. *J. Comb. Designs.* **7** (1999) 359–405.

[3] V. V. Kabanov, A. V. Mityanina, Strictly Deza line graphs. *Trudy Inst. Mat. i Mekh. UrO RAN* **18(1)** (2012) 165–177.

[4] A. V. Mityanina, On $K_{1,3}$-free strictly Deza graphs. *Trudy Inst. Mat. i Mekh. UrO RAN* **22(1)** (2016) 231–234.