On Deza graphs with parameters (v, k, k-1, a)

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joint work with Natalia Maslova and Leonid Shalaginov

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Notations

We consider undirected graphs without loops and multiple edges.

For a graph Γ and its vertex x, define the neighbourhood of x:

$$\Gamma(x) := \{y \mid y \in V(\Gamma), y \sim x\}.$$

A graph Γ is called regular of valency k if

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Definitions

A graph Δ is called a Deza graph with parameters (v, k, b, a) (where $a \leq b$), if Δ has v vertices, and for any pair of vertices $x, y \in \Delta$:

$$|\Delta(x) \cap \Delta(y)| = \begin{cases} k, & \text{if } x = y, \\ a \text{ or } b, & \text{if } x \neq y. \end{cases}$$

A graph Γ is called strongly regular with parameters (v, k, λ, μ), if Γ has v vertices, and for any pair of vertices $x, y \in \Gamma$:

$$|\Gamma(x) \cap \Gamma(y)| = \begin{cases} k, & \text{if } x = y, \\ \lambda, & \text{if } x \sim y, \\ \mu, & \text{if } x \neq y \text{ and } x \nsim y. \end{cases}$$

A Deza graph Δ is called a strictly Deza graph, if Δ has diameter 2, and is not strongly regular.

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Let Γ be a strongly regular graph with parameters (v, k, λ, μ). It's known that

(1) if $\mu = k$ then Γ is a complete multipartite graph with parts of size $v - k$;

(2) if $\mu = k - 1$ then Γ is the pentagon;

(3) if $\lambda = k - 1$ then Γ is an union of isolated cliques of size $k + 1$.

Note, cases (1) and (3) are complementary.

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Definition. Let Γ_1 and Γ_2 be graphs. A graph obtained by replacing vertices of Γ_1 by copies of Γ_2 and joining all edges between vertices from distinct copies of Γ_2 whenever the correspondent vertices of Γ_1 were adjacent is called Γ_2-extension of Γ_1.

Theorem (M. Erickson, et. al., 1999). A graph Γ is a strictly Deza graph with parameters(v, k, k, a) if and only if Γ is isomorphic to n_2-coclique extension of a strongly regular graph Γ_1 with parameters (n_1, k_1, λ, μ) for some n_1, k_1, λ, μ and n_2, where $\lambda = \mu$ and $n_2 \geq 2$.

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Main Theorem (V. K., N. Maslova, and L. Shalaginov).
A graph Γ is a strictly Deza graph with parameters $(v, k, k-1, a)$ if and only if Γ is isomorphic to 2-clique extension either of a complete multipartite graph or of a strongly regular graph with parameters $(v/2, (k-1)/2, (a-2)/2, a/2)$.

Thank you!