Maximum skew energy of tournaments

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We will consider maximum skew energy of tournaments. For a digraph D, the *skew-adjacency matrix* $S(D)$ of D is defined as $S(D) = A(D) - A(D)^T$, where $A(D)$ is the $\{0,1\}$-adjacency matrix of D and *skew energy* $\varepsilon(D)$ is defined as the sum of absolute values of eigenvalues of $S(D)$. A digraph $D = (V, E)$ is called a *tournament* if either $(x, y) \in E$ or $(y, x) \in E$ holds for any pair of $x, y \in V$ $(x \neq y)$.

For any digraph D with n vertices, $\varepsilon(D) \leq n\sqrt{n-1}$ holds [1]. Equality holds if and only if $S(D)$ is a skew symmetric conference matrix. This means that if there exists a digraph D which attains the upper bound, then n is a multiple of 4. Otherwise, we can improve this upper bound. For odd n and any digraph D with n vertices, $\varepsilon(D) \leq (n-1)\sqrt{n}$ holds. Equality holds if and only if D is a doubly regular tournament. Since $n \equiv 3 \pmod 4$ holds if there exist doubly regular tournaments with n vertices [3], there never exists a digraph D which attains the upper bound if $n \equiv 1 \pmod 4$. In both of these upper bounds, tournaments gives the maximum skew energy.

We give the upper bound of skew energy of tournaments with n vertices for $n \equiv 2 \pmod 4$ by using α-skew energy, which is the sum of the α-th power of the absolute values of the eigenvalues of a skew-adjacency matrix. For $n \equiv 2 \pmod 4$ and any tournament T, $\varepsilon(T) \leq 2\sqrt{2n-3} + (n-2)\sqrt{n-3}$ holds.

References

[1] C. Adiga, R. Balakrishnan, W. So, The skew energy of a digraph. *Linear Algebra Appl.* **432(7)** (2010) 1825–1835.

[2] A. J. Armario, M. D. Frau, Self-Dual codes from $(-1, 1)$-matrices of skew symmetric type, *arXiv:1311.2637*.

[3] P. Delsarte, J. M. Goethals, J. J. Seidel, Orthogonal matrices with zero diagonal. II. *Can. J. Math* **XXIII(5)** (1971) 816–832.

[4] G. A. Efroymson, B. Swartz, B. Wendroff, A new inequality for symmetric functions. *Adv. Math.* **38** (1980) 109–127.

[5] M. Wojtas, On Hadamard's inequality for the determinants of order non-divisible by 4. *Colloq. Math.* **12** (1964) 73–83.