Multiplicities of eigenvalues of the Star graph

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The objects of our research are spectra of Star graphs. The Star graph S_n is the Cayley graph on the symmetric group Sym_n generated by the set of transpositions $\{(1\ 2),(1\ 3),\ldots,(1\ n)\}$. In 2009 A. Abdollahi and E. Vatandoost conjectured [1] that the spectrum of S_n is integral, moreover it contains all integers in the range from $-(n-1)$ up to $n-1$ (with the sole exception that when $n \leqslant 3$, zero is not an eigenvalue of S_n). In 2012 R. Krakovski and B. Mohar [4] proved that the spectrum of S_n is integral, more precisely, they showed that for $n \geqslant 2$ and for each integer $1 \leqslant k \leqslant n$ the values $\pm(n-k)$ are eigenvalues of the Star graph S_n. They also gave a lower bound on multiplicities of eigenvalues of S_n. At the same time, G. Chapuy and V. Feray [2] showed another approach to obtain the exact values of multiplicities of eigenvalues of S_n. Their combinatorial approach is based on the Jucys–Murphy elements and the standard Young tableaux. In 2015 this approach was used to obtain the multiplicities of eigenvalues of S_n for $n \leqslant 10$ [3].

In this talk we present analytic formulas to calculate multiplicities of eigenvalues of the Star graph.

Theorem 1. *Let $n \geqslant 2$ and for each integer $1 \leqslant k \leqslant n$ the values $\pm(n-k)$ are eigenvalues of the Star graph S_n. The multiplicities $mul(n-k)$ for $k = 2,3,4,5$ of eigenvalues of S_n are given by the following formulas:*

$$mul(n-2) = (n-1)(n-2),\ n \geqslant 3; \tag{1}$$

$$mul(n-3) = \frac{(n-3)(n-1)}{2}(n^2-4n+2),\ n \geqslant 4; \tag{2}$$

$$mul(n-4) = \frac{(n-2)(n-1)}{6}(n^4-12n^3+47n^2-62n+12),\ n \geqslant 4; \tag{3}$$

$$mul(n-5) = \frac{(n-2)(n-1)}{24}(n^6-21n^5+169n^4-647n^3+1174n^2-820n+60),\ n \geqslant 5. \tag{4}$$

The following theorem gives an improved lower bound on multiplicity $mul(t)$ of eigenvalues $t := n-k$ of the Star graph which were obtained using the standard Young tableaux.

Theorem 2. *In the Star graph S_n for sufficiently large n and for a fixed t the multiplicity $mul(t)$ of eigenvalue t is at least $2^{\frac{1}{2}n\log n(1-o(1))}$.*

Thus, for any eigenvalue t of S_n the order of logarithm of multiplicities $mul(t)$ is the same that $n!$.

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References

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