Computational complexity of Vertex Cover and related problems for highly connected triangulations

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Computational complexity is studied for the Vertex Cover problem (VC) over classes of highly connected plane graphs based on Euclidean distances between their vertices (proximity graphs). Among those graphs are Delaunay triangulations and their special forms, namely half-θ_6 graphs. Study of their graph-theoretic properties received focus in the literature (Dillencourt et. al, 1996, Bose et. al 2012, Biniaz et. al, 2015). Our motivation lies in the field of network security applications as Delaunay triangulations and their relatives represent convenient network topologies which admit efficient local routing (Bose et. al 2014). More specifically VC can be considered as a problem of optimal guarding of a network where one needs to locate positions of sensors (i.e. guards) at the network nodes such that each network link is within the scope of some sensor. Being of interest VC complexity over classes of Delaunay triangulations and their relatives did not get much attention in the literature. In this work we claim VC strong NP-hardness over the class of 4-connected half-θ_6 graphs.

Planar graph is referred to as *planar triangulation* when all faces for some its plane embedding (except for possibly outer face) are triangles. Our approach to study the VC complexity for proximity graphs involves two stages. In the first one problem complexity is studied over specific classes of planar triangulations not taking into account their underlying geometric structure; in the second stage some result from graph drawing is applied that embeds graphs from these classes as Delaunay or half-θ_6 graphs.

It is known that VC is polynomially solvable over classes of outerplanar graphs (Bodlaender, 1998) including outerplanar (always Delaunay realizable due to Dillencourt, 1990) triangulations. Polynomial solvability of VC also holds true over chordal graphs (Gavril, 1972) which become triangulations when they are 3-connected and planar. More general result is known on VC polynomial solvability over class of planar graphs of bounded chordality (Kaminski, 2009). Let us observe that only a few instances of outerplanar and 3-connected planar chordal graphs are in fact 4-connected. The 3-connectivity also requires from instances of the class of planar triangulations whose chordality does not exceed k to have outer facial cycle of length not exceeding k. In our work we claim strong NP-hardness of VC over the class of 4-connected planar triangulations. Considering highly connected instances of triangulations is motivated by the fact that 4-connected (i.e. Hamiltonian) ones are graph isomorphic to Delaunay triangulations (Dillencourt, 1996). Since the proof of this isomorphism is not constructive we get weaker result on the NP-hardness of VC for 4-connected half-θ_6 graphs i.e. Delaunay triangulations build under another metric.

This work was supported by Russian Science Foundation, project 14-11-00109.