

The structure of Hentzel–Rúa semifield of order 64

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In 1991 G.P. Wene [1] has noted the hypothesis: *any finite semifield D is right or left primitive*, i.e. every element of the loop D^* is a the set of right- or left-ordered powers of an element in a semifield D . In 2004 I. Rúa [2] has indicated a counter-example to Wene’s conjecture, using Knuth’s semifield of order 32. The second counter-example is a *Hentzel–Rúa semifield* of order 64 [3], which has been constructed in 2007. The Hentzel–Rúa semifield is the unique semifield of order 64 which is neither left nor right primitive.

We investigate the structure of counter-example Hentzel–Rúa semifield and prove the conjecture that its loop is one-generated (weaker than Wene conjecture).

Lemma 1. *The automorphism group of Hentzel–Rúa semifield \mathcal{H} is isomorphic to the symmetric group S_3 and hence has exactly three involution automorphisms.*

Lemma 2. *The semifield \mathcal{H} contains exactly six maximal subfields: 5 subfields of order 8, three from them are stabilizers of different involution automorphisms; the unique subfield of order 4, which is a stabilizer of automorphism of order 3.*

Lemma 3. *The spectrum of the loop \mathcal{H}^* is $\{1, 3, 5, 6, 7\}$. The left and right spectra coincide with $\{1, 3, 6, 7, 12, 15\}$.*

Lemma 4. *For any $n \geq 10$ the loop \mathcal{H}^* is an union of all n -th degrees of any element not from maximal subfields, so \mathcal{H}^* is one-generated.*

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References

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