The structure of Hentzel–Rúa semifield of order 64

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In 1991 G.P. Wene [1] has noted the hypothesis: any finite semifield \( D \) is right or left primitive, i.e. every element of the loop \( D^* \) is a the set of right- or left-ordered powers of an element in a semifield \( D \). In 2004 I. Rúa [2] has indicated a counter-example to Wene’s conjecture, using Knuth’s semifield of order 32. The second counter-example is a Hentzel–Rúa semifield of order 64 [3], which has been constructed in 2007. The Hentzel–Rúa semifield is the unique semifield of order 64 which is neither left nor right primitive.

We investigate the structure of counter-example Hentzel–Rúa semifield and prove the conjecture that its loop is one-generated (weaker than Wene conjecture).

**Lemma 1.** The automorphism group of Hentzel–Rúa semifield \( \mathcal{H} \) is isomorphic to the symmetric group \( S_3 \) and hence has exactly three involution automorphisms.

**Lemma 2.** The semifield \( \mathcal{H} \) contains exactly six maximal subfields: 5 subfields of order 8, three from them are stabilizators of different involution automorphisms; the unique subfield of order 4, which is a stabilizer of automorphism of order 3.

**Lemma 3.** The spectrum of the loop \( \mathcal{H}^* \) is \{1, 3, 5, 6, 7\}. The left and right spectra coincide with \{1, 3, 6, 7, 12, 15\}.

**Lemma 4.** For any \( n \geq 10 \) the loop \( \mathcal{H}^* \) is an union of all \( n \)-th degrees of any element not from maximal subfields, so \( \mathcal{H}^* \) is one-generated.

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References

