Induction principle in perfect colorings theory

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Let k be a positive integer. A *k-coloring* of vertices of a graph $G = (V, E)$ is a map $f : V \to \{1, \ldots, k\}$. If $f(v) = s$ for some vertex v, then s is the *color* of v. A k-coloring of the vertices of a graph is called *perfect* if the multiset of colors of all neighbors of a vertex depends only on its own color.The induction principle is an efficient approach, that can be used for studying perfect colorings of bipartite graphs.

Consider a bipartite graph $G(V_1, V_2)$. A *half-coloring* of the graph G is a coloring of vertices of the set V_i. A half-coloring is called *feasible*, if it is a part of some perfect coloring of the graph G. Two feasible half-colorings are called *matched*, if they complement each other to the perfect coloring of the whole graph.

Let $f_1 : V_1 \to \{1, \ldots, k\}$ be an arbitrary coloring of vertices from the set V_1. *Induction operation* carries on half-coloring f_1 to coloring of the whole graph in the following way: vertices of the set V_2 with the same multiset of neighborhood colors are assigned the same color not from the set $\{1, \ldots, k\}$.

The following lemma holds:

Lemma 1. *A half-coloring of any bipartite graph G is feasible if and only if the corresponding induced coloring is perfect.*

A *bipartite coloring* is a perfect coloring of the graph G, if the color sets of the corresponding half-colorings do not intersect. Otherwise the perfect coloring of the graph G is called *nonbipartite*. Note, that color sets of half-colorings coincide in nonbipartite case, if G is connected.

The following concept is proposed to obtain the description of all perfect colorings for the graph G:

1. to obtain the complete description of feasible half-colorings of graph G using Lemma 1;
2. to construct all matched complements for each feasible half-coloring in bipartite and nonbipartite cases.

Let us call the concept described above an *induction principle*.

Authors [1] used the induction principle to obtain the complete description of perfect colorings for the infinite prism graph. It's easy to adapt results obtained in [1] to finite case - finite prism graphs and Mobius ladders.

References

[1] S. V. Avgustinovich, M. A. Lisitsyna, Perfect colorings of prism graph, to appear