Arc-transitive antipodal distance-regular covers of complete graphs: almost simple case

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We consider the problem of classification of arc-transitive antipodal distance-regular graphs of diameter three. Suppose that Γ is such a graph. Then Γ is an antipodal r-cover of K_{k+1}, Γ has intersection array $\{k, (r-1)\mu, 1; 1, \mu, k\}$, where k is the valency of Γ, r is the size of an antipodal class of Γ and μ denotes the number of common neighbours for any two vertices at distance two in Γ, and each edge of Γ lies in precisely $\lambda = k - (r-1)\mu - 1$ triangles. Let Σ be the set of antipodal classes of Γ, let $G = \mathrm{Aut}(\Gamma)$ and let $\bar{G}$ denote the permutation group, induced by G on Σ. Then $\bar{G}$ is 2-transitive on Σ. Thus, the classification of the finite 2-transitive permutation groups is crucial for the study of such graphs, and divides the problem of their description naturally up into two cases: $\bar{G}$ is almost simple or $\bar{G}$ is affine. Our aim is to study the case when the group $\bar{G}$ is almost simple. If $r \in \{2, k\}$, then Γ is distance-transitive and the classification of such graphs can be found in [1]. We also refer to [2, 3] for the case $\lambda = \mu$. Antipodal distance-regular graphs of diameter three that admit an arc-transitive action of $SU_3(q)$ have been recently classified (this result was announced in [4]). We show the following reduction theorem, which states that if $\bar{G}$ is almost simple and $\lambda \neq \mu$, then either Γ is a cover from [4], or $(soc(\bar{G}), k+1) = (L_d(q), (q^d-1)/(q-1))$, where $d \geq 3$.

Theorem. *Suppose Γ is an arc-transitive distance-regular graph with intersection array $\{k, (r-1)\mu, 1; 1, \mu, k\}$, where $r \notin \{2, k\}$, and $\lambda \neq \mu$. Let $G = \mathrm{Aut}(\Gamma)$, let Σ be the set of antipodal classes of Γ and let $\bar{G}$ denote the permutation group, induced by G on Σ. Suppose further that the socle $\bar{T}$ of the group $\bar{G}$ is a simple non-abelian group, and $(\bar{T}, k+1) \neq (L_d(q), (q^d-1)/(q-1))$, where $d \geq 3$. Then $\bar{T} = U_3(q)$, and $SU_3(q)$ acts arc-transitively on Γ with parameters $k = q^3$ and $\mu = (q+1)(q^2-1)/r$, where r divides $q+1$.*

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References

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