Hamiltonian Cycles in Directed Toeplitz Graphs

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An $n \times n$ matrix $A = (a_{ij})$ is called a Toeplitz matrix if it has constant values along all diagonals parallel to the main diagonal. A directed Toeplitz graph is a digraph with Toeplitz adjacency matrix. In this talk I will discuss conditions for the existence of hamiltonian cycles in directed Toeplitz graphs.

Notation: The main diagonal of an $n \times n$ Toeplitz adjacency matrix will be labeled 0 and it contains only zeros. The $n-1$ distinct diagonals above the main diagonal will be labeled $1, 2, \ldots, n-1$ and those under the main diagonal will also be labeled $1, 2, \ldots, n-1$. Let $s_1, s_2, \ldots, s_k$ be the upper diagonals containing ones and $t_1, t_2, \ldots, t_l$ be the lower diagonals containing ones, such that $0 < s_1 < s_2 < \cdots < n$ and $0 < t_1 < t_2 < \cdots < n$. Then, the corresponding Toeplitz graph will be denoted by $T_n < s_1, s_2, \ldots, s_k; t_1, t_2, \ldots, t_l >$. That is $T_n < s_1, s_2, \ldots, s_k; t_1, t_2, \ldots, t_l >$ is the graph with vertices $1, 2, \ldots, n$ in which the edge (i, j) occurs if and only if $j - i = s_p$ or $i - j = t_q$ for some p and q $(1 \leq p \leq k,\ 1 \leq q \leq l)$.